This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

CLASSICAL SCHOOL-BOOKS.

NOTES for LATIN LYRICS in use in Harrow, Westminster, and Rugby Schools. By the Rev. H. Musgrave Wilkins, M.A. Fellow of Merton College, Oxford. Sixth Edition. 12mo. 4*s*. 6*d*.

PROGRESSIVE EXERCISES in LATIN ELEGIAC VERSE. By Edward Walford, late Scholar of Balliol College, Oxford. Eleventh Edition of the First Series. 12mo. 2*s*. 6*d*. Key, 5*s*.

WALFORD'S PROGRESSIVE EXERCISES in LATIN ELEGIAC VERSE, Second Series; to which is prefixed a Grammar of Latin Poetry. Fourth Edition. 12mo. price 2*s*. 6*d*.

A MANUAL of GREEK PROSE COMPOSITION. By the Rev. H. Musgrave Wilkins, M.A. — Key, for Tutors and Private Students, 2*s*. 6*d*.

ELEMENTARY EXERCISES in GREEK PROSE COMPOSITION. By the Rev. H. Musgrave Wilkins, M.A. Second Edition. Crown 8vo. 4*s*. 6*d*.—Key, for Schoolmasters and Tutors only, 2*s*. 6*d*.

A PROGRESSIVE GREEK DELECTUS. By the Rev. H. Musgrave Wilkins, M.A. Fellow of Merton College, Oxford. Fifth Edition. 12mo. 4*s*.—Key, 2*s*. 6*d*.

LATIN EXERCISES for the use of the Junior Classes in King's College School, London. By the Rev. J. R. Major, D.D., Head Master. Fourteenth Edition. Fcp. 8vo. price 2*s*. 6*d*.

LATIN SELECTIONS from CORNELIUS NEPOS, PHÆDRUS, OVID'S METAMORPHOSES and EPISTLES; with Historical and Geographical Outlines, Rules of Syntax, Vocabulary, Exercises, &c. By T. S. Carr, M.A. Third Classical Master in King's College School. New Edition. Fcp. 8vo. 3*s*. 6*d*.

An ELEMENTARY LATIN GRAMMAR for the use of Schools. By the Rev. Edward Miller, M.A. late Fellow and Tutor of New College, Oxford. 12mo. 5*s*.

A SMALLER LATIN GRAMMAR for the use of Schools. Abridged from the above by the Rev. Edward Miller, M.A. 12mo. 2*s*. 6*d*.

YONGE'S LATIN GRADUS, containing every Word used by the Poets of good authority. In use in Eton, Westminster, Winchester, Harrow, Charterhouse, and Rugby Schools; King's College, London; and Marlborough College. Post 8vo. 9*s*.; *or* with Appendix, price 12*s*.

An ENGLISH-GREEK LEXICON, containing all the Greek Words used by Writers of good authority. By Charles Duke Yonge, B.A. Author of the *Latin Gradus*, as above. New Edition, being the Fifth. 4to. 21*s*.

A NEW LEXICON, ENGLISH and GREEK, abridged *for the use of Schools from* his larger 'English-Greek Lexicon' by the Author, *Charles Duke Yonge, B.A.* Square 12mo. 8*s*. 6*d*.

London: *LONGMAN*, *GREEN*, and CO. Paternoster Row.

[illegible]

600048486-

ARITHMETIC.

LONDON
PRINTED BY SPOTTISWOODE AND CO.
NEW-STREET SQUARE

ARITHMETIC

FOR THE USE OF

SCHOOLS AND STUDENTS PREPARING FOR EXAMINATION.

WITH AN APPENDIX ON

THE METRICAL SYSTEM.

BY J. FROYSELL, B.A.

TRINITY HALL, CAMBRIDGE.

LONDON:
LONGMAN, GREEN, LONGMAN, ROBERTS, & GREEN.
1864.

OTHECA · P

PREFACE.

THE AUTHOR'S object in preparing this work has been to furnish a Text-Book of Arithmetic better adapted to the wants of students preparing for the various competitive and military examinations than the works generally in use. While enabling the student to gain a thorough knowledge of the subject both in theory and practice, the Author has endeavoured to place it before him in such a manner, that the knowledge he has acquired may be *readily available* in an examination.

Great care has been taken to make the text complete, and to state the definitions, rules, and reasons for the rules, as clearly but concisely as possible; and all unnecessary and confusing explanations have been avoided. A sufficient number of carefully selected examples have been worked out to illustrate the rules, and the difficulties which most pupils meet with and frequently pass over are pointed out and explained.

At the end of the work is placed a collection of examples *systematically* arranged to correspond with the text. A great number of these have been selected from the various examination papers of the last few years, and all have been carefully chosen so as to afford sufficient practice and variety.

It is hoped that after a student has fairly mastered this work, he will experience little difficulty in answering readily, correctly, and neatly, any question he may meet with in an 'Arithmetic Paper.'

16 Montpellier Villas, Cheltenham:
November 1864.

CONTENTS.

	PAGE
Definitions, Notation, &c.	1
Greatest Common Measure	8
Least Common Multiple	10
Vulgar Fractions	11
Addition of Vulgar Fractions	15
Subtraction of Vulgar Fractions	16
Multiplication of Vulgar Fractions	17
Division of Vulgar Fractions	17
Miscellaneous Examples worked out	18
Decimal Fractions	20
Addition and Subtraction of Decimals	21
Multiplication of Decimals	22
Division of Decimals	23
To reduce a Vulgar Fraction to a Decimal	26
Recurring Decimals	27
Tables, &c.	31
Reduction	39
Compound Addition	44
Compound Subtraction	45
Compound Multiplication	46
Compound Division	48
Reduction of Fractions	50
Reduction of Decimals	52
Practice	55
Proportion	58
Single Rule *of Three*	59

PAGE
Double Rule of Three 64
Areas—Solid Contents 66
Duodecimals or Cross Multiplication 70
Interest 72
Compound Interest 75
Discount 78
Commission, Brokerage, Insurance 81
Stocks 82
Profit and Loss 85
Proportional Parts 87
Exchange 89
Equation of Payments 90
Square Root 91
Cube Root 94
Miscellaneous Examples worked out 96
Examples 100
Questions 149
Answers 153
Appendix 169

ARITHMETIC.

1. A single thing or individual is called a *unit*, and is denoted by the symbol or figure 1, *one*.

2. One unit, together with another unit of the same kind, is denoted by the symbol 2, *two*.

Two units, together with another unit of the same kind, is denoted by the symbol 3, *three*, and so on.

3. Number is the name by which we express *how many* of these units we are considering. Thus, when we say, four apples, five men, eight horses, the units are an apple, a man, a horse; and the numbers of each, four, five, eight, respectively.

4. Arithmetic is the science which treats of numbers.

5. *Notation* is the art of expressing any given number by figures.

Numeration is the art of expressing in words any number which is given in figures.

6. An *integer*, or *whole* number, is a collection of *ones* or *units*.

7. By the ordinary system of notation we are able to express *all* whole numbers by using only ten different symbols, these being the nine significant figures 1, 2, 3, 4, 5, 6, 7, 8, 9, and the cypher (0) *nought*, which has no value itself, but serves to alter the *position* of the other figures.

These ten figures are sometimes called digits.

8. The principle of this system consists in giving to the significant figures, besides their own natural value, a conventional *local* value depending on their position in the number; and the convention made is the following:—A certain place is fixed on in the number as that in which the figure occupying that place shall have its own natural value. This is called the unit's place; and each figure increases *tenfold* in value for every place it is moved to the left of the unit's place. Thus, 5 standing by itself denotes five units; but if it be moved one place to the left, as 50, it denotes ten times as many or fifty units: if moved two places to the left, it denotes one hundred times as many, and so on.

The value which thus belongs to a figure in consequence of its position is called its *local value.*

Note.—In whole numbers the unit's place is the first on the right of the number.

9. It appears, then, that in the ordinary notation we proceed towards the left from units to tens of units; from tens of units to tens of tens of units, or hundreds of units; from hundreds of units to tens of hundreds or thousands of units; from thousands of units to tens of thousands of units; thence to hundreds of thousands of units; thence to thousands of thousands, which are called millions of units; thence to tens of millions, hundreds of millions, &c., of units, till we come to millions of millions, which are called billions of units; and so on to trillions, quadrillions, &c.

10. When no specific value is assigned to the units of which a number is composed, it is called an *abstract number.* Thus five, seven, are abstract numbers, their units having no specific value.

When some specific value is assigned to the units of which a number is composed, it is called a *concrete number.* Thus, five horses, seven men, are concrete numbers, the *units now having* the specific values—a horse, a man, respectively.

11. The following signs and abbreviations are used in arithmetic:—

$+$ (*plus*) denotes that the number before which it stands is to be added to the preceding number.

$-$ (*minus*) denotes that the number before which it stands is to be subtracted from the preceding number.

$\times$ (*into*) denotes that the numbers between which it stands are to be multiplied together.

$\div$ (*by*) placed between two numbers denotes that the former is to be divided by the latter.

$=$ (*equal*) denotes that the numbers between which it stands are equal to one another.

$\therefore$ is used instead of the word *therefore*.

$\because$ is used instead of the words *since* or *because*.

12. When two or more numbers are added together, the result is called their *sum*.

When one number is subtracted from another, the result is called their *difference*.

When one number is multiplied by another, the former is called the *multiplicand*, the latter the *multiplier*, and the result is called the *product*. When three or more numbers are multiplied together, the result is called their *continued product*.

When one number is divided by another, the former is called the *dividend*, the latter the *divisor*, and the result is called the *quotient*.

These four, Addition, Subtraction, Multiplication, and Division, are the fundamental operations of arithmetic; but it must be observed that the two last are only convenient methods of performing addition and subtraction.

13. When a number is multiplied by itself any number of times the result is called a *power* of that number, and is briefly expressed by writing down the number with a small figure denoting how many times it is repeated a little above it on the right. Thus, $2^2=2\times2=4$; $7^3=7\times7\times7=343$; $10^4=10\times10\times10\times10=10000$.

14. The second power is called the *square* of the number, the third power its *cube*. Thus, $49=7^2$ is the square of 7; $64=4^3$ is the cube of 4.

15. The square root of a number is that number which, when multiplied by itself, will produce the given number, and is denoted by the symbol $\sqrt{\ }$: thus, $\sqrt{49}=7\ \because 7\times7$ or $7^2=49$. $\sqrt{121}=11$: $\because 11\times11$ or $11^2=121$.

The cube root of a number is that number which, when multiplied by itself twice, will produce the given number, and is denoted by the symbol $\sqrt[3]{\ }$. Thus, $\sqrt[3]{27}=3\ \because 3\times3\times3$ or $3^3=27$; $\sqrt[3]{1000}=10 \because 10\times10\times10$ or $10^3=1000$.

16. A number which is the product of two or more numbers all greater than unity is called a *Composite Number*, and the numbers which, when multiplied together, produce it are called *Factors*. Thus, $10=2\times5$; $42=2\times3\times7$ are composite numbers, their factors being 2 and 5; 2, 3, and 7 respectively.

17. A number which cannot be separated into factors is called a *Prime Number*, thus, 5, 7, 13 are prime numbers.

18. Numbers which, although not prime numbers themselves, have no common factor, are said to be prime to each other; thus, 14 and 15 are prime to each other; so also are 12 and 25.

19. It will be found useful to remember that a number is always divisible—

(1) by 2 when the last figure is divisible by 2.
(2) „ 4 when the two last figures are divisible by 4.
(3) „ 8 when the three last figures are divisible by 8.
(4) „ 5 when the last figure is 5 or 0.
(5) „ 3 or 9 when the sum of the figures is divisible by 3 or 9.

(6) by 6 or 12 when the sum of the figures is divisible by 3, and the last or the two last figures by 2 and 4 respectively.

(7) „ 11 when the difference between the sum of the figures in the odd places and the sum of the figures in the even places is either 0 or is divisible by 11.

20. (1) Every figure except the last in a number is a number of tens, and 10 is divisible by 2: it therefore depends on the last figure whether the whole number be divisible by 2 or not.

(2) Every figure except the two last in a number is a number of hundreds; and 100 is divisible by 4; it therefore depends on the two last figures whether the whole number be divisible by 4 or not.

(3) Every figure except the three last in a number is a number of thousands, and 1000 is divisible by 8: it therefore depends on the three last figures whether the whole number be divisible by 8 or not.

(4) Every figure except the last in a number is a number of tens, and 10 is divisible by 5: also since 5 is the only number less than 10 which is divisible by 5, it follows that the whole number will be so divisible only when it ends in 5 or 0.

(5) Since $10=9+1$; $100=99+1$; $1000=999+1$; $20=9\times2+2$; $200=99\times2+2$; $2000=999\times2+2$, we see that the remainder left after dividing 1, 10, 100, 1000, &c., by 3 or 9 is 1; that the remainder left after dividing 2, 20, 200, 2000, &c., by 3 or 9 is 2, and so on.

If then we divide any number, say 2764, by 9 in parcels, 2000 will be 2 more than an exact number of nines, 700 will be 7 more, and 60 will be six more. So then from 2, 7, 6, 4 added together, and the nines taken out, comes the only remainder which can come from 2764.

(6) This follows from (5) together with (1) or (2).

(7) We observe that $10+1$, $100-1$, $1000+1$, $10000-1$,

&c., 20+2, 200−2, 2000+2, 20,000−2, &c., are all divisible by 11, and so on, and from this the rule given may be deduced.

Ex. 1.—Required the remainder of 246120377 divided by 9.

The sum of the figures is 2 + 4 + 6 + 1 + 2 + 0 + 3 + 7 + 7, or 32, which gives the remainder 5. But the easiest way is to throw out the nines as fast as they arise in the sum. Thus, repeat 2, 6, 12 (2 + 4 + 6), say 3 (throwing out 9), 4, 6, 9 (throw this away), 7, 14 (and throwing out the 9), 5, which is the remainder required.

Ex. 2.—Is 8792361 divisible by 9?

Casting out nines, we have 8, 15, *6*, 8, 11, *2*, 8, 9, *0*. There is no remainder, and we therefore know that 8792361 is divisible by 9.

Ex. 3.—Is 3675842104 divisible by 11?

The sum of the figures in the even places is 3 + 7 + 8 + 2 + 0, or 20. The sum of those in the odd places is 6 + 5 + 4 + 1 + 4, or 20; hence the difference is 0, and we therefore know that 3675842104 is divisible by 11.

21. Casting out nines is also very useful in testing the correctness of multiplication. To do this cast nines out of the multiplicand and multiplier. Multiply the remainders together, and again cast out nines. If the multiplication be correct this last remainder will be the same as the remainder left by casting nines out of the product.

Ex. 1.

$$\begin{array}{rlrl}
528 & \ldots & 6 & \\
\underline{345} & \ldots & \underline{3} & \\
2640 & & 18 & — 0 \\
2112 & & & \\
\underline{1584} & & & \\
\underline{182160} & \ldots\ldots & & 0
\end{array}$$

Ex. 2.

$$\begin{array}{rlrl}
326 & \ldots & 2 & \\
\underline{35} & \ldots & \underline{8} & \\
1630 & & 16 & — 7 \\
\underline{978} & & & \\
\underline{11410} & \ldots\ldots & & 7
\end{array}$$

This test will show if we are *wrong*, but not always that we are right; for if we have omitted 9 or any multiple of 9, or misplaced figures, these errors will not be detected.

22. Since the dividend = the quotient × by the divisor, we may employ the same test in division, remembering to subtract the remainder (if there be any) from the dividend. We then proceed as in multiplication, considering the dividend as the product.

23. It may here be mentioned that the shortest way to multiply by 5 is to annex a cypher and divide by 2. To divide by 5, multiply by 2 and cut off the last figure, which leaves the quotient: half the last figure is the remainder.

To multiply by 25 annex two cyphers and divide by 4. To divide by 25, multiply by 4 and cut off the two last figures, which leaves the quotient: one-fourth of the two last figures is the remainder.

To multiply by 125 annex three cyphers and divide by 8. To divide by 125 multiply by 8 and cut off the three last figures, which leaves the quotient: one-eighth of the three last figures is the remainder.

To multiply by 9 annex a cypher and then subtract each figure from the preceding.

To multiply by 11 annex a cypher and add each figure to the preceding.

To multiply by any number between 12 and 20 in one line, multiply as usual by the units of the multiplier and add to each successive product, besides the number carried from the last product, the preceding figure in the multiplicand.

The truth of these rules is obvious, since 5 is the half of ten; 25 the fourth of 100; 125 the eighth of 1000; 9 is $10-1$; 11 is $10+1$; while the last amounts only to adding in at once the product of the multiplicand by 10, instead of first writing it down in a separate line.

$$
\begin{array}{r}
13572\\
17\\
\hline
230724\\
\hline
\end{array}
$$

Ex. 1.

Here we have $2\times 7=14$, carry 1; $7\times 7=49$ and 1 are 50, and 2 from

the multiplicand are 52, carry 5 ; 5 × 7 = 35 and 5 are 40, *and* 7 *from the multiplicand* are 47, carry 4; 3 × 7 = 21, and 4 are 25, *and* 5 *from the multiplicand* are 30, carry 3; 1 × 7 = 7 and 3 are 10, *and* 3 *from the multiplicand* are 13, carry 1; 1 *and* 1 *from the multiplicand* are 2.

24. The system of notation explained in Art. 8 was brought into Europe by the Arabs, and is therefore often called the Arabic Notation. The following was the notation used by the Romans.

I stood for 1, V for 5, X for 10, L for 50, C for 100, D or IↃ for 500, M or CIↃ for 1000.

When any character was *followed* by one of less or equal value the expression denoted the sum of their values. Thus, III for stood 3, VI for 6, XX for 20, LXVII for 67.

When any character was *preceded* by one of less value the expression denoted the difference of their values. Thus, IV stood for 4, IX for 9, XL for 40.

Every Ↄ annexed to IↃ increased its value tenfold. Thus IↃↃ stood for 5000, IↃↃↃ for 50000, and so on.

Every C and Ↄ joined to CIↃ increased its value tenfold. Thus, CCIↃↃ stood for 10000, CCCIↃↃↃ for 100000, and so on.

A line drawn over a character increased its value a thousandfold. Thus $\overline{V}$ stood for 5000, $\overline{IX}$ for 9000, $\overline{C}$ for 100000, and so on.

GREATEST COMMON MEASURE.

Any number which will divide a number without remainder is said to be a *measure* of it.

25. *A Common Measure* of two or more numbers is a number which will divide each of them without remainder. Thus 3 is a common measure of 12, 18, 24. So also is 2.

The Greatest Common Measure of two or more numbers is the *greatest* number which will divide each of them without remainder. Thus 6 is the greatest common measure (G. C. M.) of 12, 18, 24.

26. *To find the G. C. M. of two numbers.*

Rule.—Divide the greater number by the less: this divisor by the remainder, and so on, each divisor becoming in turn the dividend, and each remainder the divisor, until there is no remainder. The last divisor will be the G. C. M. required.

Ex. 1. Find the G. C. M. of 6409 and 7395.

```
6409)7395(1
     6409
      986)6409(6
          5916
           493)986(2
               986
               ...
```

∴ 493 is the G. C. M. required.

Ex. 2. Find the G. C. M. of 2023 and 7581.

```
2023)7581(3
     6069
     1512)2023(1
          1512
           511)1512(2
               1022
                490)511(1
                    490
                     21)490(23
                        42
                         70
                         63
                          7)21(3
                            21
                            ...
```

∴ 7 is the G. C. M. required.

27. *To find the G. C. M. of three or more numbers.*

Rule.—Find the G. C. M. of the two first numbers; then the G. C. M. of this and the third number, and so on. The last G. C. M. so found will be the one required.

LEAST COMMON MULTIPLE.

28. Every number which contains another number as a factor (Art. **16**), or, in other words, which is exactly divisible by that number, is called a ***multiple*** of it. Thus 6 is a multiple of 2 and also of 3.

A *Common Multiple* of two or more numbers is a number which is a multiple of each of them. Thus 24 is a common multiple of 2, 3, and 4.

The *Least Common Multiple* (L. C. M.) of two or more numbers is the *least* number which is a multiple of each of them. Thus 24 is the L. C. M. of 2, 3, 4, 6, and 8.

29. *To find the L. C. M. of two or more numbers.*

Rule.—Write the given numbers in a line from left to right, and then strike out those which are contained in any of the others. Divide the numbers which remain by any number which will exactly divide more than one of them. Set the quotients under the corresponding dividends, and bring down unaltered those numbers into which this divisor will not exactly go. In the new line thus formed again strike out those which are contained in any of the others, and proceed as before until the numbers left in the last line are all prime to each other. The continued product of these and the several divisors will be the L. C. M. required.

Ex. 1. Find the L. C. M. of 6, 15, 24, 25.

5)~~6~~, 15, 24, 25
~~3~~, 24, 5 *Ans.* 600.

Here 6 is contained in 24 and is therefore struck out. Dividing by 5 the new line is 3, 24, 5; 3 is contained in 24 and is therefore struck out; 24 and 5 are prime to each other; and 5 × 24 × 5 = 600 the L. C. M. required.

Ex. 2. Find the L. C. M. of 6, 8, 9, 14, 18, 36, 42, 56, 63.

7)~~6~~, ~~8~~, ~~9~~, ~~14~~, ~~18~~, 36, 42, 56, 63
4)36, ~~6~~, 8, ~~9~~
9, 2 *Ans.* 504.

Here 6, 9, and 18 are contained in 36; 8 and 14 in 56, these are therefore struck out. Dividing by 7 the new line is 36, 6, 8, 9; 6 and 9 are contained in 36 and are therefore struck out. Dividing by 4 the new line is 9 and 2; 9 and 2 are prime to each other; and $2 \times 9 \times 4 \times 7 = 504$ the L. C. M. required.

VULGAR FRACTIONS

30. A *Fraction* is a part or parts of a unit, and is expressed by two numbers, written one above the other, with a line between them. Thus $\frac{2}{3}$ denotes that the unit is to be divided into three equal parts of which two are to be taken; $\frac{11}{17}$ denotes that the unit is to be divided into seventeen equal parts of which eleven are to be taken.

The lower number is called the *denominator*, because it denotes into how many parts the unit is to be divided, and the upper one the *numerator*, because it tells how many of those parts are to be taken.

A fraction also represents the quotient of the numerator divided by the denominator.

31. A *Proper Fraction* is one whose numerator is less than the denominator. Thus, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{9}{13}$ are proper fractions.

An *Improper Fraction* is one whose numerator is equal to or greater than the denominator. Thus $\frac{4}{4}$, $\frac{5}{3}$, $\frac{7}{2}$, $\frac{12}{5}$ are improper fractions.

A *Mixed Number* is one made up of a whole number and a fraction. Thus, $1\frac{3}{4}$, $7\frac{9}{13}$ are mixed numbers.

A *Compound Fraction* is a fraction of a fraction. Thus, $\frac{3}{5}$ of $\frac{7}{8}$, $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{3}{7}$ are compound fractions.

A *Complex Fraction* is one which has a fraction or a mixed number for its numerator, or for its denominator, or for both. Thus, $\frac{\frac{5}{7}}{9}$, $\frac{5}{1\frac{7}{13}}$, $\frac{1\frac{3}{5}}{\frac{24}{25}}$ are complex fractions.

32. *To multiply a fraction by a whole number.*

Rule.—Either multiply the numerator or divide the denominator by the whole number. Thus $\frac{3}{7}\times 2=\frac{6}{7}$, for in each of the fractions the unit is divided into seven equal parts, and twice as many of those parts are taken in the latter case as in the former.

So, also, $\frac{3}{8}\times 2=\frac{3}{4}$, for in the former of these fractions the unit is divided into eight equal parts, and in the latter into four: each of these latter parts is therefore equal to two of the former; and, since the same number of parts are taken, the latter fraction is twice as great as the former.

Note.—The second method should be used whenever the denominator is exactly divisible by the multiplier.

33. *To divide a fraction by a whole number.*

Rule.—Either divide the numerator or multiply the denominator by the whole number. Thus, $\frac{6}{7}\div 2=\frac{3}{7}$, for in each of the fractions the unit is divided into seven equal parts, but only half as many of those parts are taken in the latter case as in the former. So, also, $\frac{3}{4}\div 2=\frac{3}{8}$, for in the former of these fractions, the unit is divided into four equal parts, and in the latter into eight; each of these latter parts is, therefore, half one of the former, and since the same number of parts are taken, the latter fraction is half the former.

Note.—The first method should be used whenever the numerator is exactly divisible by the divisor.

34. If the numerator and denominator of a fraction be both multiplied by the same number the value of the fraction will not be altered. For, by multiplying the numerator, the fraction is multiplied (Art. **32**), and by multiplying the denominator, it is divided (Art. **33**); and the value of a quantity is not altered if it be both multiplied and divided by the same number. Similarly, the value of a fraction will not be altered if the numerator and denominator be both divided by the same number.

35. Since every whole number may be considered as a fraction whose denominator is 1, it follows from Art. **34** that we may express a whole number as a fraction with any denominator we please by taking for the numerator the product of the whole number multiplied by the given denominator. Thus, to express 5 as a fraction with denominator 7, we have $5=\frac{5}{1}=\frac{5\times7}{1\times7}=\frac{35}{7}$: so also 11 expressed as a fraction with the denominator 12, gives $\frac{132}{12}$.

36. A mixed number, as $2\frac{3}{5}$, denotes 2 units together with $\frac{3}{5}$ of a unit, and 2 units $=\frac{10}{5}$ of 1 unit (Art. **35**); it follows, therefore, that $2\frac{3}{5}$ is equivalent to $\frac{10}{5}$ of a unit together with $\frac{3}{5}$ of a unit, that is, to $\frac{13}{5}$ of a unit, or $\frac{5\times2+3}{5}$. And conversely, since $\frac{13}{5}$ denotes that the unit is to be divided into 5 equal parts, and 13 such parts taken; but 10 such parts make up exactly 2 units; therefore $\frac{13}{5}$ of a unit is equivalent to 2 units together with $\frac{3}{5}$ of a unit, that is, to $2\frac{3}{5}$.

Hence we see that every mixed number is equivalent to an improper fraction, and may be expressed as such by multiplying the whole number by the denominator of the fraction, and adding in the numerator for the new numerator, retaining the same denominator. Thus $7\frac{1}{2}=\frac{15}{2}$, $13\frac{5}{7}=\frac{96}{7}$. And conversely, every improper fraction is equivalent to a mixed number, and may be expressed as such by dividing the numerator by the denominator. The quotient will be the whole number, and the remainder (if any) the numerator of the fraction retaining the same denominator. Thus, $\frac{9}{4}=2\frac{1}{4}$, $\frac{27}{8}=3\frac{3}{8}$.

37. A fraction is said to be in its *lowest terms* when the numerator and denominator are prime to each other (Art. **18**).

38. *To reduce a fraction to its lowest terms.*

RULE.—Divide the numerator and denominator by their greatest common measure (Art. **26**).

This will not alter the value of the fraction (Art. **34**).

In practice it is frequently unnecessary to find the G. C. M. of the numerator and denominator, as we can see by inspection (Art. **19**) the factors which are common to both, and dividing by these successively, the fraction will be reduced to its lowest terms.

Ex. 1. Reduce $\frac{7501}{8655}$ to its lowest terms.

The G. C. M. of 7501 and 8655 will be found to be 577, and dividing the numerator and denominator by 577 we have $\frac{7501}{8655}=\frac{13}{15}$.

Ex. 2. Reduce $\frac{3888}{5616}$ to its lowest terms.

$$\begin{aligned}\tfrac{3888}{5616}&=\tfrac{432}{624}\text{ (dividing by 9)}\\&=\tfrac{108}{156}\text{ (dividing by 4)}\\&=\tfrac{9}{13}\text{ (dividing by 12).}\end{aligned}$$

And since 9 and 13 are prime to each other, the fraction is now in its lowest terms.

39. *To reduce a compound fraction to a simple one.*

RULE.—Find the continued product of the numerators for the new numerator, and of the denominators for the new denominator.

Note 1.—Mixed numbers must be reduced to improper fractions (Art. **36**) before applying the rule.

Note 2.—Any factors which are common to one of the numerators and one of the denominators should be struck out before applying the rule; the resulting fraction will then be in its lowest terms.

Ex. 1. $\frac{3}{5}$ of $\frac{4}{7}=\dfrac{3\times4}{5\times7}=\frac{12}{35}$.

For $\frac{1}{5}$ of $\frac{4}{7}=\frac{4}{35}$ (Art. **33**) $\therefore$ $\frac{3}{5}$ of $\frac{4}{7}=\frac{4}{35}\times3=\frac{12}{35}$ (Art. **32**).

Ex. 2. $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{5}{8}=\dfrac{5\times3\times5}{7\times4\times8}=\frac{75}{224}$.

Ex. 3. $\frac{3}{7}$ of $\frac{14}{15}$ of $\frac{5}{8}$ of $\frac{3}{4}=\dfrac{\cancel{3}}{\cancel{7}}$ of $\dfrac{\overset{\cancel{2}}{\cancel{14}}}{\underset{\cancel{5}}{\cancel{15}}}$ of $\dfrac{\cancel{5}}{\underset{4}{\cancel{8}}}$ of $\dfrac{3}{4}=\dfrac{3}{16}$.

40. *To reduce fractions to equivalent fractions having a common denominator.*

Rule.—Find the L. C. M. (Art. **29**) of all the denominators; take this for the common denominator; and for the new numerators multiply each numerator by the quotient obtained by dividing the common denominator by its own denominator.

This amounts to multiplying the numerator and denominator of each fraction by the same number. Its value, therefore, is not altered (Art. **34**).

When fractions are reduced to a common denominator their values can be compared at once.

Ex. 1. Reduce $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{18}$ to their least common denominator.

The L. C. M. of the denominators, 4, 6, 8, 18, is 72, and by the rule we have $\frac{3 \times 18}{72}, \frac{5 \times 12}{72}, \frac{7 \times 9}{72}, \frac{11 \times 4}{72}$ $\therefore$ the fractions required are $\frac{54}{72}, \frac{60}{72}, \frac{63}{72}, \frac{44}{72}$.

Here, in the first fraction, the denominator 4 was multiplied by 18 in order to produce 72, and, therefore, that the value of the fraction may not be altered, the numerator 3 must also be multiplied by 18, and similarly for the other fractions.

Ex. 2. Which is the greatest and which the least of the fractions $\frac{5}{7}, \frac{9}{16}, \frac{25}{28}$?

Reducing them to their least common denominator, we have $\frac{80}{112}, \frac{63}{112}, \frac{100}{112}$; therefore $\frac{25}{28}$ is the greatest, and $\frac{9}{16}$ the least.

41. *Addition of fractions.*

Rule.—Reduce the fractions to equivalent ones with their least common denominator. Take the sum of the new numerators for the numerator, retaining the common denominator.

Note 1.—If there be any whole or mixed numbers the integral parts should be added together separately.

Note 2.—Compound fractions must be reduced to simple ones, and improper fractions should be reduced to mixed numbers before applying the rule.

Ex. 1. Add together $\frac{3}{5}, \frac{5}{8}, \frac{1}{6}, \frac{7}{10}$.

$$\tfrac{3}{5}+\tfrac{5}{8}+\tfrac{1}{6}+\tfrac{7}{10}=\frac{72+75+20+84}{120}=\tfrac{251}{120}=2\tfrac{11}{120}.$$

Ex. 2. Add together $2\frac{2}{3}, 3\frac{3}{4}, 4\frac{4}{5}, 5\frac{5}{6}$.

$$2\tfrac{2}{3}+3\tfrac{3}{4}+4\tfrac{4}{5}+5\tfrac{5}{6}=2+3+4+5+\tfrac{2}{3}+\tfrac{3}{4}+\tfrac{4}{5}+\tfrac{5}{6}=14+\frac{40+45+48+50}{60}=$$

$$14+\tfrac{183}{60}=14+3\tfrac{3}{60}=17\tfrac{1}{20}.$$

Ex. 3. Find the sum of $2\frac{1}{3}+1\frac{3}{4}+\frac{3}{7}$ of $\frac{14}{15}$ of $\frac{7}{8}+\frac{11}{12}$.

Reducing the compound fraction to a simple one we have $\frac{\cancel{3}}{\cancel{7}}$ of $\frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{15}}}$ of $\frac{7}{\underset{4}{\cancel{8}}}=\frac{7}{20}$, and $2\frac{1}{3}+1\frac{3}{4}+\frac{7}{20}+\frac{11}{12}=3+\frac{1}{3}+\frac{3}{4}+\frac{7}{20}+\frac{11}{12}=3+\frac{20+45+21+55}{60}=3$ $+\frac{141}{60}=3+2\frac{21}{60}=5\frac{7}{20}$.

42. *Subtraction of fractions.*

RULE.—Reduce the fractions to equivalent ones with their least common denominator. Take the difference of these numerators for the new numerator, retaining the common denominator.

Note 1.—If there be any whole or mixed numbers, the integral parts should be subtracted separately.

Note 2.—Compound fractions must be reduced to simple ones, and improper fractions should be reduced to mixed numbers, before applying the rule.

Ex. 1. Subtract $\frac{3}{14}$ from $\frac{6}{7}$.

$$\tfrac{6}{7}-\tfrac{3}{14}=\frac{12-3}{14}=\tfrac{9}{14}.$$

Ex. 2. Subtract $2\frac{5}{12}$ from $3\frac{7}{8}$.

$$3\tfrac{7}{8}-2\tfrac{5}{12}=3-2+\tfrac{7}{8}-\tfrac{5}{12}=1+\frac{21-10}{24}=1\tfrac{11}{24}.$$

Ex. 3. Subtract $2\frac{3}{8}$ from $4\frac{1}{4}$.

$$4\tfrac{1}{4}-2\tfrac{3}{8}=2+\tfrac{1}{4}-\tfrac{3}{8}=2+\frac{2-3}{8}=2-\tfrac{1}{8}=1+1-\tfrac{1}{8}=1\tfrac{7}{8}.$$

Here, since the second fraction is greater than the first, their difference $\frac{1}{8}$ must be subtracted from the whole number. Or, which is easier in practice, take 1 from the whole number, and subtract the fraction from it: the difference between unity and a proper fraction being found at once by subtracting the numerator of the fraction from the denominator. Thus, $1-\frac{3}{7}=\frac{4}{7}$.

Ex. 4. Subtract $2\frac{7}{8}$ from $5\frac{5}{12}$.

$$5\frac{5}{12}-2\frac{7}{8}=3+\frac{5}{12}-\frac{7}{8}=3+\frac{10-21}{24}=3-\frac{11}{24}=2+1-\frac{11}{24}=2\frac{13}{24}.$$

43. *Multiplication of fractions.*

Rule.—Find the continued product of the numerators for the new numerator, and of the denominators for the new denominator.

It will be observed that this rule is the same as that given in Art. **39**, for the reduction of a compound fraction to a simple one.

When the multiplier is a whole number, the term multiplication signifies taking some multiple of a quantity; but when the multiplier is a fraction, the term is used in a more extended sense, and signifies taking some *part* or some *multiple of a part* of a quantity. Thus $\frac{2}{3}\times\frac{4}{5}$ denotes that a fifth part of the quantity represented by $\frac{2}{3}$ is to be taken four times, and the operation is therefore the same as taking $\frac{4}{5}$ of $\frac{2}{3}$.

Ex. 1. Find the product of $1\frac{5}{7}\times3\frac{1}{2}\times\frac{11}{12}$.

$$\frac{12}{7}\times\frac{7}{2}\times\frac{11}{12}=\frac{\cancel{12}}{\cancel{7}}\times\frac{\cancel{7}}{2}\times\frac{11}{\cancel{12}}=\frac{11}{2}=5\frac{1}{2}.$$

44. *Division of fractions.*

Rule.—Invert the divisor and multiply.

The term *division*, when applied to fractions, signifies finding that quantity which, when multiplied by the divisor, will produce the dividend.

Notes 1 and 2 of Art. **39** must be observed in multiplication and division of fractions. If the divisor be itself a compound fraction, each of the fractions composing it must be inverted before applying the rule.

Ex. 1. Divide $\frac{4}{5}$ by $\frac{8}{15}$.

$$\frac{4}{5}\div\frac{8}{15}=\frac{\cancel{4}}{\cancel{5}}\times\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{8}}}=\frac{3}{2}=1\frac{1}{2}.$$

Ex. 2. Divide $2\frac{3}{5}$ of $\frac{5}{7}$ by $\frac{6}{7}$ of $4\frac{1}{2}$.

$$2\tfrac{3}{5} \text{ of } \tfrac{5}{7} \div \tfrac{6}{7} \text{ of } 4\tfrac{1}{2} = \tfrac{13}{5} \text{ of } \tfrac{5}{7} \times \tfrac{7}{6} \times \tfrac{2}{9} = \frac{13}{\cancel{5}} \text{ of } \frac{\cancel{5}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{3}{\cancel{6}}} \times \frac{\cancel{2}}{9} = \tfrac{13}{27}.$$

45. *Miscellaneous Examples worked out.*

Ex. 1. Find the value of $\frac{3}{7} + 1\frac{5}{14} + 2\frac{1}{3}$ of $1\frac{4}{21}$ of $\frac{4}{5} + 2\frac{1}{6}$.

$$2\tfrac{1}{3} \text{ of } 1\tfrac{4}{21} \text{ of } \tfrac{4}{5} = \frac{\cancel{7}}{3} \text{ of } \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{21}}} \text{ of } \frac{4}{\cancel{5}} = \tfrac{20}{9} = 2\tfrac{2}{9}.$$

$$\tfrac{3}{7} + 1\tfrac{5}{14} + 2\tfrac{2}{9} + 2\tfrac{1}{6} = 5 + \frac{54 + 45 + 28 + 21}{126} = 5 + \tfrac{148}{126} = 5 + 1\tfrac{22}{126} = 6\tfrac{11}{63}.$$

Ex. 2. Find the value of $8\frac{2}{3} - 3\frac{5}{8}$ of $1\frac{7}{9} + 2\frac{3}{5}$ of $2\frac{3}{26} - 1\frac{7}{12}$.

$$3\tfrac{5}{8} \text{ of } 1\tfrac{7}{9} = \frac{29}{\cancel{8}} \text{ of } \frac{\overset{2}{\cancel{16}}}{9} = \tfrac{58}{9} = 6\tfrac{4}{9};$$

$$2\tfrac{3}{5} \text{ of } 2\tfrac{3}{26} = \frac{\cancel{13}}{\cancel{5}} \text{ of } \frac{\overset{11}{\cancel{55}}}{\underset{2}{\cancel{26}}} = \tfrac{11}{2} = 5\tfrac{1}{2};$$

$$8\tfrac{2}{3} - 6\tfrac{4}{9} + 5\tfrac{1}{2} - 1\tfrac{7}{12} = 6 + \tfrac{2}{3} - \tfrac{4}{9} + \tfrac{1}{2} - \tfrac{7}{12} = 6 + \frac{24 - 16 + 18 - 21}{36} =$$

$$6 + \frac{42 - 37}{36} = 6\tfrac{5}{36}.$$

Ex. 3. Find the value of $7\frac{3}{5} - \frac{5}{8} - 3\frac{5}{12} + 3\frac{7}{30} - \frac{15}{16}$.

$$7\tfrac{3}{5} - \tfrac{5}{8} - 3\tfrac{5}{12} + 3\tfrac{7}{30} - \tfrac{15}{16} = 7 + \tfrac{3}{5} - \tfrac{5}{8} - \tfrac{5}{12} + \tfrac{7}{30} - \tfrac{15}{16}$$

$$= 7 + \frac{144 - 150 - 100 + 56 - 225}{240} = 7 + \frac{200 - 475}{240} = 7 - \tfrac{275}{240}$$

$$= 7 - 1\tfrac{35}{240} = 6 - \tfrac{7}{48} = 5 + 1 - \tfrac{7}{48} = 5\tfrac{41}{48}.$$

Ex. 4. Simplify $\dfrac{1\frac{5}{7} + 2\frac{3}{4} - 3\frac{7}{12}}{2\frac{10}{21} - 1\frac{5}{6} + \frac{7}{8}}$.

$$\frac{1\frac{5}{7} + 2\frac{3}{4} - 3\frac{7}{12}}{2\frac{10}{21} - 1\frac{5}{6} + \frac{7}{8}} = \frac{\frac{5}{7} + \frac{3}{4} - \frac{7}{12}}{1 + \frac{10}{21} - \frac{5}{6} + \frac{7}{8}} = \frac{\dfrac{60 + 63 - 49}{84}}{1 + \dfrac{80 - 140 + 147}{168}} = \frac{\frac{74}{84}}{1\frac{87}{168}} = \frac{74}{\cancel{84}} \times \frac{\overset{2}{\cancel{168}}}{255} = \tfrac{148}{255}.$$

Ex. 5. Simplify $2 + \dfrac{1\frac{2}{3} + \frac{3}{5}}{1\frac{2}{3} - \frac{3}{5}} + \dfrac{1\frac{7}{9}}{8} - \dfrac{9}{3\frac{3}{5}}$.

$$2 + \frac{1\frac{2}{3} + \frac{3}{5}}{1\frac{2}{3} - \frac{3}{5}} + \frac{1\frac{7}{9}}{8} - \frac{9}{3\frac{3}{5}} = 2 + \frac{1 + \dfrac{10 + 9}{15}}{1 + \dfrac{10 - 9}{15}} + \frac{\frac{16}{9}}{8} - \frac{9}{\frac{18}{5}} = 2 + \frac{1\frac{19}{15}}{1\frac{1}{15}} + \frac{\overset{2}{\cancel{16}}}{9} \times \frac{1}{\cancel{8}} - \cancel{9} \times \frac{5}{\underset{2}{\cancel{18}}}$$

$$=2+\frac{\overset{17}{\cancel{34}}}{\cancel{15}}\times\frac{\cancel{15}}{\underset{8}{\cancel{16}}}+\tfrac{2}{9}-2\tfrac{1}{2}=2+2\tfrac{1}{8}+\tfrac{2}{9}-2\tfrac{1}{2}=2\frac{9+16-36}{72}=2-\tfrac{11}{72}=1\tfrac{61}{72}.$$

Ex. 6. Simplify $\left\{\tfrac{3}{7}\text{ of }\dfrac{\frac{5}{4}\text{ of }\frac{14}{15}}{\frac{9}{5}\div\frac{18}{5}}+\tfrac{7}{2}\div\dfrac{\frac{7}{8}-\frac{1}{2}}{\frac{3}{7}+\frac{1}{2}}-\dfrac{\frac{21}{16}\div\frac{7}{4}}{\frac{8}{5}\times\frac{5}{32}}\right\}\div 6\tfrac{2}{3}$

$$\left\{\tfrac{3}{7}\text{ of }\dfrac{\frac{5}{4}\text{ of }\frac{14}{15}}{\frac{9}{5}\div\frac{18}{5}}+\tfrac{7}{2}\div\dfrac{\frac{7}{8}-\frac{1}{2}}{\frac{3}{7}+\frac{1}{2}}-\dfrac{\frac{21}{16}\div\frac{7}{4}}{\frac{8}{5}\times\frac{5}{32}}\right\}\div\tfrac{20}{3}$$

$$=\left\{\tfrac{3}{7}\text{ of }\dfrac{\frac{7}{8}}{\frac{1}{2}}+\tfrac{7}{2}\div\dfrac{\frac{3}{8}}{\frac{13}{14}}-\dfrac{\frac{3}{4}}{\frac{1}{4}}\right\}\times\tfrac{3}{20}.$$

$$=\left\{\frac{\cancel{3}}{\cancel{7}}\text{ of }\frac{\cancel{7}}{\underset{2}{\cancel{8}}}\times\cancel{2}+\frac{\cancel{7}}{\cancel{2}}\times\frac{\overset{2}{\cancel{8}}}{3}\times\frac{13}{\cancel{14}}-\frac{3}{\cancel{4}}\times\cancel{4}\right\}\times\tfrac{3}{20}$$

$$=\left\{1+8\tfrac{2}{3}-3\right\}\times\tfrac{3}{20}=6\tfrac{2}{3}\times\frac{3}{20}=\tfrac{20}{3}\times\tfrac{3}{20}=1.$$

Ex. 7. Simplify $2+\cfrac{2}{3+\cfrac{\frac{1}{3}}{2+\cfrac{\frac{3}{5}}{4+\frac{1}{5}}}}$

$4+\frac{1}{5}=\frac{21}{5}$, $\frac{3}{5}\times\frac{5}{21}=\frac{1}{7}$, $2+\frac{1}{7}-\frac{15}{7}$, $\frac{1}{3}\times\frac{7}{15}=\frac{7}{45}$, $3+\frac{7}{45}=\frac{142}{45}$, $2\times\frac{45}{142}=\frac{45}{71}$, $2+\frac{45}{71}$ $=2\frac{45}{71}$.

Ex. 8. What fraction added to $\frac{3}{5}+1\frac{2}{3}$ of $1\frac{4}{5}$ of $\frac{7}{9}$ will give $3\frac{1}{2}$?

$$\tfrac{3}{5}+\frac{\cancel{5}}{3}\text{ of }\frac{\cancel{9}}{\cancel{5}}\text{ of }\frac{7}{\cancel{9}}=\tfrac{3}{5}+2\tfrac{1}{3}=2\tfrac{14}{15}.$$

We know that $3\frac{1}{2}$ is the sum of $2\frac{14}{15}$, and the required number, which will therefore be given by $3\frac{1}{2}-2\frac{14}{15}=1+\frac{1}{2}-\frac{14}{15}=1+\dfrac{15-28}{30}=1-\frac{13}{30}=\frac{17}{30}$. Hence $\frac{17}{30}$ is the fraction required.

Ex. 9. What fraction subtracted from $2\frac{5}{7}$ will leave $\frac{4}{5}$?

We know that $2\frac{5}{7}$ is the sum of the remainder $\frac{4}{5}$, and the required number, which will therefore be given by $2\frac{5}{7}-\frac{4}{5}=2+\dfrac{25-28}{35}=2-\frac{3}{35}$ $=1\frac{32}{35}$. Hence $1\frac{32}{35}$ is the fraction required.

Ex. 10. What fraction multiplied by $4\frac{2}{3}$ will produce $5\frac{3}{5}$?

Since the product is $5\frac{3}{5}$, and one factor $4\frac{2}{3}$, the required factor must be given by $5\frac{3}{5}\div4\frac{2}{3}$. Hence $\frac{28}{5}\times\frac{3}{14}=1\frac{1}{5}$ is the fraction required.

Ex. 11. What fraction divided by $3\frac{1}{3}$ will produce $5\frac{3}{20}$?

Since the dividend is the product of the divisor and the quotient, the required number must be $3\frac{1}{3}\times5\frac{3}{20}=\frac{10}{3}\times\frac{103}{20}=\frac{103}{6}=17\frac{1}{6}$.

DECIMAL FRACTIONS.

46. It was shown in Art. **8** how we can express all whole numbers by the ordinary system of notation; and if we extend the system in the same manner to figures standing to the right of the units place, each figure decreasing tenfold in value for every place it occupies to the right of this, we shall be able to express with a uniform notation numbers less than unity or fractions. Thus the first figure to the right of the units place will have one-tenth of its natural value, the second one-hundredth, and so on.

47. The units place is denoted by a point, called the decimal point, placed to its right. The figures standing to the right of this point are called Decimals or Decimal Fractions, and the number of them is called the number of Decimal places. Thus ·875 is a decimal of three places.

48. It appears, then, that $\cdot 8=\frac{8}{10}$, $\cdot 87=\frac{8}{10}+\frac{7}{100}=\frac{87}{100}$ or $\frac{87}{10^2}$, $\cdot 875=\frac{8}{10}+\frac{7}{100}+\frac{5}{100}=\frac{875}{1000}$ or $\frac{875}{10^3}$, and so on. Hence we see that a decimal may be expressed as a vulgar fraction, having 10 or some power of 10 for its denominator.

49. From the last article it appears that, when a decimal is expressed as a vulgar fraction, the number of cyphers in the denominator is the same as the number of decimal places. Hence a given decimal may at once be expressed as a vulgar fraction by writing down the decimal for the numerator, and for the denominator 1, followed by as many cyphers as there were decimal places.

Conversely, a vulgar fraction having any power of 10 for its denominator may at once be expressed as a decimal by marking off from the numerator as many decimal places as there are cyphers in the denominator. Thus $78\cdot 75=78\frac{75}{100}$, $3\cdot 1436=3\frac{1436}{10000}$, $\cdot 0425=\frac{425}{10000}$; also $\frac{564}{1000}=\cdot 564$, $\frac{10565}{10000}=1\cdot 0565$, $\frac{13}{10000}=\cdot 0013$.

Note.—It is evident that cyphers placed to the *right* of a decimal (like those placed to the left of a whole number) do not alter its value. Thus $\cdot80=\frac{8}{10}+\frac{0}{100}=\cdot8$, $\cdot1300=\frac{1}{10}+\frac{3}{100}+\frac{0}{1000}+\frac{0}{10000}=\cdot13$.

50. *A decimal, then, may be defined as—*

(i) A fraction expressed according to the ordinary system of notation; or

(ii) A fraction expressed by digits, each digit having a local value dependent on its position; or

(iii) A fraction whose denominator is 10 or some power of 10.

51. Since $87\cdot5\times10=\frac{875}{10}\times10=875\cdot$, $8\cdot75\times100=\frac{875}{100}\times100=875\cdot$, $\cdot5875\times1000=\frac{5875}{10000}\times1000=\frac{5875}{10}=587\cdot5$.

We see that a decimal is multiplied by 10, 100, 1000, &c., by moving the decimal point one, two, three, &c., places respectively to the *right.*

Conversely, since $875\cdot\div10=\frac{875}{10}=87\cdot5$, $875\cdot\div100=\frac{875}{100}=8\cdot75$, $587\cdot5\div1000=\frac{5875}{10}\times\frac{1}{1000}=\frac{5875}{10000}=\cdot5875$, $3\cdot85\div10000=\frac{385}{100}\times\frac{1}{10000}=\frac{385}{1000000}=\cdot000385$.

We see that a decimal is divided by 10, 100, 1000, &c., by moving the decimal point one, two, three, &c., places respectively to the *left.*

52. *Addition and subtraction of decimals.*

Rule.—Write the numbers one under each other, as in common addition and subtraction, being careful to keep figures of the same denomination in the same column; units under units, tens under tens, one-tenths under one-tenths, &c. Add or subtract as with whole numbers, and place the decimal point in the sum under the decimal point above.

Note.—Cyphers may be added to fill up vacant places to the right of the decimals.

Ex. 1. Add together 589·37, 7·326, 24·5, ·13564 and ·0003.

$$\begin{array}{r} 589{\cdot}37000 \\ 7{\cdot}32600 \\ 24{\cdot}50000 \\ {\cdot}13564 \\ {\cdot}00030 \\ \hline 621{\cdot}33194 \\ \hline \end{array}$$

Ex. 2. Subtract 5·9937 from 55·008.

$$\begin{array}{r} 55{\cdot}0080 \\ 5{\cdot}9937 \\ \hline 49{\cdot}0143 \\ \hline \end{array}$$

53. The reason for this rule is obvious; the figures in each column being all of the same denomination. Thus the first column to the right of the units consists of the numerators of fractions whose denominators are all 10, and which may therefore be added or subtracted at once (Arts. **41, 42**), and similarly for the other columns.

54. *Multiplication of decimals.*

Rule.—Multiply the numbers together as if they were whole numbers, and mark off in the product as many decimal places as there are in the multiplicand and multiplier together.

Ex. 1. Multiply 6·0305 by 3·02; also ·0005 by ·03.

$$\begin{array}{r} 6{\cdot}0305 \\ 3{\cdot}02 \\ \hline 120610 \\ 180915 \\ \hline 18212110 \end{array} \qquad \textit{Ans. } 18{\cdot}21211. \qquad\qquad \begin{array}{r} {\cdot}0005 \\ {\cdot}03 \\ \hline 15 \\ \hline \end{array} \quad \textit{Ans. } {\cdot}000015.$$

55. The reason for this rule will appear from the following:

$$6{\cdot}0305 \times 302 = \frac{60305}{10000} \times \frac{302}{100} = \frac{60305 \times 302}{1000000} = \text{a}$$

fraction whose numerator is the product of the two numbers and denominator 1, followed by as many cyphers as there were decimal places in the multiplicand, + as many as there were decimal places in the multiplier. This fraction, when expressed as a decimal, will therefore have as many decimal places as there are in the multiplicand and multiplier together. Hence the rule.

56. *Division of decimals.*

Rule.—Divide the given numbers as if they were whole numbers: then—

If the number of decimal places in the divisor be the *same* or *greater* than the number in the dividend, the quotient will be a *whole* number, and in the latter case as many cyphers must be annexed to it as make up the difference between the number of decimal places in the divisor and dividend; but—

If the number of decimal places in the divisor be *less* than the number in the dividend, as many *decimal places* must be marked off from the quotient as make up the difference.

If necessary, cyphers must be added to the right of the decimal, but any so added must be counted as decimal places in determining the position of the decimal point in the quotient.

Ex. 1. Divide 32455·28 by 5·72.

5·72)32455·28(5674

2860

3855

3432

4232

4004

2288

2288 *Ans.* 5674.

Here the number of decimal places in the divisor and dividend is the same. The quotient is therefore a whole number.

Ex. 2. Divide 2147·04 by ·036.

4)214704

9)53676

5964 *Ans.* 59640.

Here the number of decimal places in the divisor is one more than the number in the dividend. We must therefore annex one cypher to the quotient, which will be a whole number.

Ex. 3. Divide 124·9974 by 23·54.

$$\begin{array}{l}
23{\cdot}54)124{\cdot}9974(531\\
\underline{11770}\\
7297\\
\underline{7062}\\
2354\\
\underline{2354}
\end{array}$$

Ans. 5·31.

Here the number of decimal places in the divisor is two less than the number in the dividend. We therefore mark off two decimal places in the quotient.

Ex. 4. Divide 3217 by ·0625.

$$\begin{array}{l}
625)3217(51472\\
\underline{3125}\\
920\\
\underline{625}\\
2950\\
\underline{2500}\\
4500\\
\underline{4375}\\
1250\\
\underline{1250}
\end{array}$$

Ans. 51472.

Here we have to annex four cyphers to the dividend, and counting these as decimal places, we have the same number in the divisor and dividend. The quotient is therefore a whole number.

Ex. 5. Divide 4·63638 by 81·34.

$$\begin{array}{l}
81{\cdot}34)4{\cdot}63638(57\\
\underline{40670}\\
56938\\
\underline{56938}
\end{array}$$

Ans. ·057.

Here the number of decimal places in the divisor is three less than the number in the dividend. We therefore mark off three decimal places in the quotient.

57. The reason for these rules will appear from the following :—

(1) $1{\cdot}44 \div {\cdot}12 = \frac{144}{100} \times \frac{100}{12} = \frac{144}{12} = 12.$

(2) $14{\cdot}4 \div {\cdot}12 = \frac{144}{10} \times \frac{100}{12} = \frac{144}{12} \times 10 = 120.$

(3) ${\cdot}144 \div 1{\cdot}2 = \frac{144}{1000} \times \frac{10}{12} = \frac{144}{12} \times \frac{1}{100} = {\cdot}12.$

From (1) we see that when the number of decimal places is the same in both, the quotient is a whole number.

From (2) that the quotient must be multiplied by 10, 100, 1000, &c.; that is, must have one, two, three, &c., cyphers annexed to it if the number of decimal places in the divisor be one, two, three, &c., more than the number in the dividend.

From (3) that the quotient must be divided by 10, 100, 1000, that is, must have one, two, three, &c., decimal places if the number of decimal places in the divisor be one, two, three, &c., less than the number in the dividend.

58. The following rule may also be used for division of decimals:

Rule.—If the number of places in the divisor and dividend be different, annex as many cyphers to the one which has the less number of places as will make the number the same in both, and divide as if they were whole numbers. The decimal point in the quotient will stand before the first figure, which requires *another* cypher to be added to the dividend.

Ex. 1. Divide 71·712 by ·83.

$$\begin{array}{r} \cdot830)71\cdot712(86\cdot4 \\ 66\,40 \\ \hline 5\,312 \\ 4\,980 \\ \hline 3320 \\ 3320 \\ \hline \end{array}$$

Ans. 86·4.

Here we make the number of decimal places the same by annexing a cypher to the divisor, and in order to obtain the complete quotient have to add one cypher to the dividend; there will therefore be one place of decimals in the quotient.

Ex. 2. Divide 3·1 by ·0025.

$$\begin{array}{r} \cdot0025)3\cdot1000(1240 \\ 2\,5 \\ \hline 60 \\ 50 \\ \hline 100 \\ 100 \\ \hline 0 \\ \hline \end{array}$$

Ans. 1240.

O

Here we add three cyphers to the dividend and the divisor will go exactly 1240 times into the number so formed.

Ex. 3. Divide ·011214 by 53·4.

53·400000)·011214 0000(·00021
10680 0000
534 0000
534 0000

Ans. ·00021.

Here to make the number of decimal places the same we annex five cyphers to the divisor, but the number so formed will not go into the dividend; we therefore add a cypher to the latter, it still will not go, we therefore get ·0 in the quotient and proceed as before.

59. *To reduce a vulgar fraction to a decimal.*

We have seen (Art. **59**) that when the denominator of a vulgar fraction is some power of 10, the fraction may at once be expressed as a decimal by marking off from the numerator as many places as there are cyphers in the denominator.

If, however, the denominator be not a power of 10, we must try if we can find a factor which will make it one, for then multiplying the numerator and denominator by this factor, we obtain a fraction in the required form. Now since the required power of 10 in this case is the product of the denominator and the required factor, that factor will be found by dividing the power of 10 by the denominator. If, therefore, we divide the denominator into 1, followed by as many cyphers as necessary for decimal places, and multiply the numerator by the quotient, we shall obtain the decimal required, or, which amounts to the same thing, and is more convenient in practice,

RULE.—Divide the numerator of the fraction by the denominator, annexing as many cyphers as necessary for decimal places. The quotient will be the decimal required.

Ex. 1. Reduce $\frac{17}{125}$, $\frac{83}{16}$ to decimals.

125)17·0(13·6
125
450
375
·750
750

$\therefore \frac{17}{125} = \cdot 136$

16)83·(5·1875
80
30
16
140
128
120
112
80
80

$\therefore \frac{83}{16} = 5 \cdot 1875$.

60. Since 10 contains only the factors 2 and 5, if the denominator of a fraction in its lowest terms contain any other factor besides these, there is no number which, when multiplied by the denominator, will produce an exact power of 10, such a fraction, therefore, cannot be expressed *exactly* as a decimal. If, however, we proceed according to the rule in the last article, although the division will never terminate, we may, by taking a sufficient number of places, make the decimal so obtained differ *as little as we please* from that of the fraction and may therefore regard it as its true value.

61. Again, when one number is divided by another, the remainder must always be less than the divisor, and therefore the number of *different* remainders cannot exceed *one less than the number* expressed by the divisor. Hence at some point in the division the same remainder must come over again, and therefore in reducing a vulgar fraction to a decimal, since the figure brought down is always the same, namely, 0,. all the subsequent remainders must recur also in the same order, and therefore the same figures in the quotient.

Such decimals are called *circulating*, *repeating*, or *recurring decimals*.

The figures which circulate or form the recurring part are called the *period*, which is marked by a point placed over the first and last figures composing it.

62. On the whole, then, it appears that a vulgar fraction in its lowest terms will produce a finite decimal when its denominator contains no factor other than 2 and 5 or their powers, and that it will produce a recurring decimal if its denominator contain any factor other than 2 and 5 or their powers.

That the greatest possible number of figures in the period is one less than the number expressed by the denominator of the fraction.

Thus, $\frac{27}{80}=\frac{27}{2^4\times5}$; $\frac{9}{25}=\frac{9}{5^2}$; $\frac{1}{64}=\frac{1}{2^6}$ will all produce finite decimals, but $\frac{29}{30}=\frac{29}{2\times5\times3}$; $\frac{17}{84}=\frac{17}{2^2\times3\times7}$; $\frac{6}{13}$ will all produce recurring decimals, and the number of figures in each period cannot exceed 29, 83, and 12, respectively.

Ex. 1. Reduce $\frac{17}{84}$ and $\frac{368}{495}$ to decimals.

84)17·0(202 38095
168
200
168
320
252
680
672
800
756
440
420
20

Ans. $\cdot 20\dot{2}3809\dot{5}$.

495)368·0(743
346 5
21 50
19 80
1 700
1 485
215

Ans. $\cdot 7\dot{4}\dot{3}$.

Here the remainders 20 and 215 recur. ∴ $\frac{17}{84}=\cdot 20\dot{2}3809\dot{5}$, and $\frac{368}{495}=\cdot 7\dot{4}\dot{3}$.

Ex. 2. Reduce $\frac{1}{7}$ and $\frac{1}{11}$ to decimals.

$$7)\underline{1\cdot000000} \qquad\qquad 11)\underline{1\cdot00}$$
$$\cdot142857 \text{ \&c.} \qquad\qquad \cdot09 \text{ \&c.}$$

$\therefore \frac{1}{7} = \cdot\dot{1}4285\dot{7}$, and $\frac{1}{11} = \cdot\dot{0}\dot{9}$.

When, as in the two last fractions, the period begins immediately after the decimal point, it is called a *pure* circulator. When this is not the case, as in the two first fractions, it is called a *mixed* circulator.

63. *To reduce a recurring decimal to a vulgar fraction.*

Rule.—Write down the decimal to the end of the first period, and subtract from it the figures (if any) which do not recur.

Take this number for the numerator, and for the denominator write down as many *nines* as there are recurring figures, followed by as many cyphers as there are non-recurring figures.

The fraction so formed, reduced to its lowest terms, will be the fraction required.

Without some knowledge of algebra the student can hardly understand the reason for this rule.

Ex. 1. Reduce $\cdot\dot{9}2\dot{7}$, $\cdot06\dot{5}\dot{1}$ and $4\cdot791\dot{6}$ to vulgar fractions.

$$\cdot\dot{9}2\dot{7} = \frac{927}{999} = \frac{103}{111}.$$

$$\cdot06\dot{5}\dot{1} = \frac{651-6}{9900} = \frac{645}{9900} = \frac{129}{1980} = \frac{43}{660}.$$

Here we have two recurring and two non-recurring figures, and subtracting the latter, have 645 for the numerator, while the denominator is 9900.

$$4\cdot791\dot{6} = 4\frac{7916-791}{9000} = 4\frac{7125}{9000} = 4\frac{19}{24}.$$

Here we have one recurring and three non-recurring figures, and subtracting the latter, have 7125 for the numerator, while for the denominator we have one *nine* followed by three cyphers.

64. By this rule $\cdot\dot{9} = \frac{9}{9} = 1$; $\cdot\dot{9}\dot{9} = \frac{99}{99} = 1$; $\cdot\dot{9}9\dot{9} = \frac{999}{999} = 1$ and so on.

But $1 - \cdot9 = \cdot1$; $1 - \cdot99 = \cdot01$; $1 - \cdot999 = \cdot001$, and so on;

hence we see that the difference between 1 and the decimal becomes less and less as we increase the number of decimal places, and although this difference will never actually vanish, we may make it *as small as we please.*

It is in this sense that we say a recurring decimal is equal to a vulgar fraction, the difference between them becoming less than any assignable quantity.

65. When circulating decimals are to be added or subtracted, and only a certain number of places are required to be correct, it will generally be sufficient to carry out the decimal to three or four places more than are required, taking care to add 1 to the last figure retained if the next figure be 5 or greater than 5.

In all operations, however, involving multiplication or division by recurring decimals, they must first be reduced to vulgar fractions, and the required operation having been performed, the result if necessary can be reduced back to a decimal.

Ex. 1. Find the value correct to 5 places of decimals of $2\cdot\dot{4}1\dot{8}+1\cdot1\dot{6}+3\cdot\dot{0}0\dot{9}+\cdot735\dot{4}+24\cdot042$; and of $7\cdot\dot{7}\dot{2}-6\cdot0\dot{4}\dot{5}$.

$$\begin{array}{r} 2\cdot41841841 \\ 1\cdot16666666 \\ 3\cdot00900900 \\ \cdot73544444 \\ 24\cdot04200000 \\ \hline 31\cdot37153851 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 7\cdot7272727 \\ 6\cdot0454545 \\ \hline 1\cdot6818182 \\ \hline \end{array}$$

Ans. 1·68182.

Ans. 31·37154.

In each of these instances, since the first figure rejected is 8, the last figure retained is increased by 1.

Ex. 2. Multiply $1\cdot8\dot{3}$ by $3\cdot\dot{6}\dot{3}$.

$$1\cdot8\dot{3}=1\frac{83-8}{90}=1\tfrac{75}{90}=1\tfrac{5}{6}=\tfrac{11}{6};$$

$$3\cdot\dot{6}\dot{3}=3\tfrac{63}{99}=3\tfrac{7}{11}=\tfrac{40}{11};$$

$$\frac{\cancel{11}}{\underset{3}{\cancel{6}}}\times\frac{\overset{20}{\cancel{40}}}{\cancel{11}}=\tfrac{20}{3}=6\cdot\dot{6}.$$

$\therefore 6\cdot\dot{6}$ is the answer required.

Ex. 3. Divide $3{\cdot}41\dot{6}$ by $8{\cdot}2$.

$$8{\cdot}2=8\tfrac{1}{5}=\tfrac{41}{5};\ 3{\cdot}41\dot{6}=3\frac{416-41}{900}=3\tfrac{375}{900}=3\tfrac{5}{12}=\tfrac{41}{12};\ \tfrac{41}{12}\times\tfrac{5}{41}=\tfrac{5}{12}=41\dot{6}.$$

TABLES.

66. MONEY.

4 farthings	make	1 penny (*d.*)
12 pence	„	1 shilling (*s.*)
20 shillings	„	1 pound (£)

Pounds, shillings, and pence are denoted by the letters *£ s. d.*, these being the first letters of the Latin words *libra*, *solidus*, and *denarius*. Farthings are usually written as fractions of a penny. Thus $\frac{1}{4}$*d.*, $\frac{1}{2}$*d.* (=$\frac{2}{4}$*d.*), and $\frac{3}{4}$*d.*

The following coins have been in use in England at different times; but those marked (0) are now obsolete, except the *groat*, *tester*, and *angel*, which are used under different names.

(0)	A groat	=	4*d.*	(0)	A half-guinea	=	10*s.* 6*d.*
(0)	A tester	=	6*d.*	(0)	A mark	=	13*s.* 4*d.*
	A florin	=	2*s.* 0*d.*		A sovereign	=	£1 0*s.* 0*d.*
	A half-crown	=	2*s.* 6*d.*	(0)	A guinea	=	£1 1*s.* 0*d.*
	A crown	=	5*s.* 0*d.*	(0)	A carolus	=	£1 3*s.* 0*d.*
(0)	A noble	=	6*s.* 8*d.*	(0)	A jacobus	=	£1 5*s.* 0*d.*
(0)	An angel	=	10*s.* 0*d.*	(0)	A moidore	=	£1 7*s.* 0*d.*

The first five are silver coins, and the rest gold.

67. The standard *gold coin* of this country consists of 22 parts of pure gold and 2 parts of alloy. From a lb. troy of this metal are coined $46\frac{29}{40}$ sovereigns=£46 14*s.* 6*d.* The mint price of standard gold is therefore $\frac{1}{12}$ of £46 14*s.* 6*d.*=£3 17*s.* $10\frac{1}{2}$*d.*

The standard silver coin of this country consists of 37 parts of pure silver and 3 parts of alloy. From a lb. troy of this metal are coined 66*s.* The mint price per oz. of standard silver is therefore $\frac{1}{12}$ of 66*s.*=5*s.* 6*d.*

Copper coin is not a legal tender for more than 12*d.*, nor is silver for more than 40*s.*: the gold coinage being the standard in this country.

68. TROY WEIGHT.

24 grains	make	1 pennyweight (dwt.)
20 pennyweights	„	1 ounce (oz.)
12 ounces	„	1 pound (lb.)

This weight is probably so called from Troyes, a city in France, where large fairs were once held. It is used in weighing gold and silver, jewels, and the like.

It was ordered by an Act of Parliament which came into force January 1, 1826, that the *brass weight of one pound troy of the year* 1758 should be the unit or *standard measure of weight*, that all other weights should be derived from it, and that 5760 *grains* should be contained in the pound troy. Now, the weight of a *cubic inch* of distilled water, the barometer being at 30 inches, and Fahrenheit's thermometer at 62°, is known to be 252·458 such grains, and therefore the Imperial Standard Pound can at any time be recovered if it should be lost or injured. The Brass Standard Pound was, in fact, rendered useless by the fire at the House of Commons in 1834.

69. Precious stones are weighed by *carats*.

A carat (of diamond) = $3\frac{1}{6}$ grains
A carat (of gold or silver) = 240 grains

Since there are 24 carats in a pound of gold, the quality of gold is expressed by saying it is so many *carats fine*, that is, that so many parts out of 24 are pure gold. Thus standard gold is 22 carats fine, and jewellers' gold 18 carats fine.

70. AVOIRDUPOIS WEIGHT.

16 drams	make	1 ounce (oz.)
16 ounces	„	1 pound (lb.)
28 pounds	„	1 quarter (qr.)
4 quarters	„	1 hundred-weight (cwt.)
20 hundred-weights	„	1 ton (ton)

This weight probably derives its name from the old Norman *avoirs*, goods or chattels, and *pois*, weight, and is used in weighing all heavy goods, as meat, grocery, &c., in almost all commercial transactions, and in the common dealings of life.

The pound avoirdupois contains 7000 grains troy.

The following weights are also used:—

A stone = 14 lbs.
but for meat or fish = 8 lbs.
A firkin of butter = 56 lbs.
A truss of straw = 36 lbs.
A truss of old hay = 56 lbs.
A truss of new hay = 60 lbs.
A load of hay or straw = 36 trusses
A great pound (of silk) = 24 oz.
A pack of wool = 240 lbs.
A fodder of lead = 19½ cwt.

71. Apothecaries' Weight.

20 grains	make	1 scruple (℈)
3 scruples	„	1 dram (ʒ)
8 drams	„	1 ounce (℥)
12 ounces	„	1 pound (lb.)

Apothecaries mix their medicines by this weight; but buy and sell their drugs by avoirdupois weight.

The pound and ounce are the same as the pound and ounce troy, but differently subdivided.

72. Linear Measure.

12 inches	make	1 foot (ft.)
3 feet	„	1 yard (yd.)
5½ yards	„	1 rod, pole, or perch (po.)
40 poles	„	1 furlong (fur.)
8 furlongs	„	1 mile (m.)
3 miles	„	1 league (lea.)

It was ordered by the Act of Parliament referred to in Art. **68** that the *Brass Standard Yard* of 1760 should be the unit or standard measure of length, that all other measures of length should be derived from it, and that the *thirty-sixth* part of this yard should be an *inch*. Now, the length of a pendulum which will vibrate seconds in a vacuum, in the latitude of London and at the level of the sea, is known to be 39·1393 such inches, and therefore the Imperial Standard Yard can at any time be recovered if it should be lost or injured.

The following measures are also used:—

A line	=	$\frac{1}{12}$ of an inch	A cubit	=	18 inches
A palm	=	3 inches	A pace	=	5 feet
A hand	=	4 inches	A fathom	=	6 feet
A span	=	9 inches			

A chain used in measuring land is 22 yards, or 100 links.

73. CLOTH MEASURE.

2¼ inches	make	1 nail
4 nails	„	1 quarter
4 quarters	„	1 yard
5 quarters	„	1 English ell
6 quarters	„	1 French ell
3 quarters	„	1 Flemish ell

74. SQUARE MEASURE.

144 square inches	make	1 square foot (s. ft.)
9 square feet	„	1 square yard (s. yd.)
30¼ square yards	„	1 square pole (P.)
40 square poles	„	1 rood (R.)
4 roods	„	1 acre (A.)

This measure is used to measure all kinds of surfaces.

A square inch, foot, &c. is the area of a square, each of whose sides is an inch, foot, &c.

The following measures are also used :—

A square of flooring	=	100 s. ft.
A rod of brickwork	=	272¼ s. ft.
10 square chains, or 100000 square links	=	1 acre
A yard of land	=	30 acres
A hide of land	=	100 acres

75. Cubic Measure.

1728 cubic inches make 1 cubic foot (c. ft.)
27 cubic feet „ 1 cubic yard (c. yd.)

A cubic inch, foot, &c. is the solid content of a cube or figure contained by six equal squares, each of whose edges is a linear inch, foot, &c.

The following measures are also used :—

A load of rough timber = 40 c. ft.
A load of hewn timber = 50 c. ft.
A ton of shipping = 42 c. ft.

76. Wine Measure.

4 gills	make	1 pint (pt.)
2 pints	„	1 quart (qt.)
4 quarts	„	1 gallon (gal.)
10 gallons	„	1 anker (ank.)
18 gallons	„	1 runlet (run.)
42 gallons	„	1 tierce (tier.)
2 tierces	„	1 puncheon (pun.)
63 gallons	„	1 hogshead (hhd.)
2 hogsheads	„	1 pipe (pipe)
2 pipes	„	1 tun (tun)

77. Ale and Beer Measure.

2 pints	make	1 quart (qt.)
4 quarts	„	1 gallon (gal.)
9 gallons	„	1 firkin (fir.)
18 gallons	„	1 kilderkin (kil.)
36 gallons	„	1 barrel (bar.)
1½ barrels or 54 gallons	„	1 hogshead (hhd.)
2 hogsheads	„	1 butt (butt)
2 butts	„	1 tun (tun)

78. Corn or Dry Measure.

2 quarts	make	1 pottle (pot.)
2 pottles	„	1 gallon (gal.)
2 gallons	„	1 peck (pk.)
4 pecks	„	1 bushel (bus.)
2 bushels	„	1 strike (str.)
4 bushels	„	1 coomb (coomb)
2 coombs or 8 bushels	„	1 quarter (qr.)
5 quarters	„	1 load (ld.)
2 loads or 10 quarters	„	1 last (last)

79. The standard measure of capacity for liquids and dry goods is the imperial gallon. This contains 277·274 cubic inches, which is the space exactly filled by 10 pounds avoirdupois weight of distilled water, weighed in air at a temperature of 62°, Fahrenheit's thermometer, the barometer being at 30 inches.

80. NUMBER.

12 units	make	1 dozen
12 dozen	„	1 gross
20 units	„	1 score
120 units	„	1 long hundred
24 sheets of paper	„	1 quire
20 quires	„	1 ream
10 reams	„	1 bale

81. TIME.

60 seconds	make	1 minute
60 minutes	„	1 hour
24 hours	„	1 day
7 days	„	1 week
4 weeks	„	1 lunar month
365 days	„	1 year

The year is also divided into 12 *calendar* months. The number of days in each month are easily remembered by means of the following lines.

Thirty days hath September,
April, June, and November,
February hath twenty-eight alone,
And all the rest have thirty-one,
But Leap-year, coming once in four,
February then has one day more.

82. The *mean solar day* which is the standard unit for the measurement of *time*, is the mean or average time which elapses between two successive transits of the sun across the meridian of any place.

The *solar year* is the time which elapses between the sun's leaving and returning to a certain fixed point in the ecliptic, and consists of 365.242218 mean solar days, or 365 days, 5 hrs., 48 min., 47½ sec., very nearly.

The civil year which consists of 365 days is, consequently, very nearly ¼ of a day less than the solar year, and if this were not allowed for, the error would accumulate, and in the course of time cause great confusion. To prevent this, Julius Cæsar ordered that every fourth year should consist of 366 days. This was called *leap* or *bissextile year*, and the extra day, called the *intercalary day*, was taken in in February, which then had twenty-nine days instead of twenty-eight.

This correction, however, was too great, since the Julian year consisted of 365·25 days, and the error 365·25 — 365·242218, or ·007782 of a day, would in time become considerable. Hence it happened that the Vernal Equinox, which, at the Council of Nice, A.D. 325, fell on March 21, in the year A.D. 1582 fell on March 11; the error in 1257 years having amounted to $\cdot007782 \times 1257 = 9\cdot7819$ or nearly 10 days. To correct this, Pope Gregory ordered that 10 days should be omitted in that year, making October 15 to follow October 4, and to prevent the recurrence of this error, he ordered that 3 leap years should be omitted in every 400 years, viz. those which complete a century, when the number of *hundreds* is not divisible by 4, because in that time the error would amount to $\cdot007782 \times 400 = 3\cdot1128$, or a little more than 3 days.

The error left by this correction is ·000282 of a day in each year, and will only amount to a day in 4000 years.

Obs. All years are leap years when the two last figures of the date are divisible by 4, *except those which complete a century*, and then, in order that the year may be a leap year, the two *first* figures must be divisible by 4. Thus 1756, 1848, 1864, 1600, 2000, are leap years, but not 1810, 1875, 1700, 2200, &c.

The Julian calculation, which is called the *old style*, is still used in Russia. The Gregorian, or *new style*, was adopted in England A.D. 1752, when the error had amounted to 11 days.

83. It should be observed that the mean solar day is the standard from which all our measures and weights are derived. The $\frac{1}{86400}$th part of a day is a second. From the pendulum which will vibrate seconds we derive the standard yard (Art. **72**), from this the measures of surface and solidity, and from the last of these we obtain the pound, the standard measure of weight (Art. **68**), and the gallon, the standard measure of capacity (Art. **79**).

84. In France all the measures and weights are derived from the *metre* which is the $\frac{1}{10000000}$th part of the terrestrial arc from the equator to the pole, and=39·371 inches. The other measures are all decimal parts or multiples of this. Thus the decimetre=3·9371 in., the centimetre=·39371 in., and so the decametre=393·71 in., and so on.

The standard of weight is the *gramme*, which is the weight of a cubic centimetre of distilled water, and= 15·444 grs.; the other weights are decimal parts or multiples of this.

The standard of capacity is the *litre*, which contains 61·028 c. in., and the other measures are as before decimal parts or multiples of this.

The adoption of this, which is called the metrical system in England, is, at the present time, very warmly advocated by some; but while the advantages which it possesses over our own system are very questionable, the confusion and inconvenience the change would occasion is so great that the general feeling of the country appears to be decidedly against it.

85. The question of a decimal coinage for this country has been agitated for some years. If adopted we should have the pound sterling as at present, the florin=$\frac{1}{10}$ of a pound, the cent$=\frac{1}{10}$ of a florin$=\frac{1}{100}$ of a pound, and the mil$=\frac{1}{10}$ of a cent$=\frac{1}{100}$ of a florin$=\frac{1}{1000}$ of a pound.

Our money table would then be as follows:—

10 mils	make	1 cent (c.)
10 cents	„	1 florin (fl.)
10 florins	„	1 pound (£)

If this decimal system were adopted, much of the labour of reducing quantities from one denomination to another would be done away with, since they might be written at once as decimals, and all arithmetical operations performed with them as such. Thus £18 3fl. 9m.=£18·309; 24372m. =2437·2 cents=243·72fl.=£24·372, or £24 3fl. 7c. 2m.

REDUCTION.

86. Reduction is the name given to the method by which we express a quantity in a denomination different from that in which it is given.

To reduce a quantity to a lower denomination.

Rule.—Multiply the highest denomination in the proposed quantity by the number of units of the next lower denomination which make one of the highest, adding into the product the number (if any) of that next lower denomination in the proposed quantity: and proceed in the same manner for each succeeding denomination until the one required is arrived at.

Ex. 1. Reduce £1027 17*s.* 6¼*d.* to farthings.

£ *s.* *d.*
1027 17 6¼
20
20557 = number of shillings in £1027 + 17*s.*
12
246690 = number of pence in 20557*s.* + 6*d.*
4
986761 = number of farthings in 986761*d.* + 1*f.*

986761. *Ans.*

Ex. 2. Reduce 17 tons 19 cwt. 1 qr. 12 lbs. 10 oz. to drams.

tons cwt. qr. lbs. oz.
17 19 1 12 10
20
359 = number of cwts. in 17 tons + 19 cwt.
4
1437 = number of quarters in 359 cwt. + 1 qr.
28
11508
2874
40248 = number of lbs. in 1437 qrs. + 12 lbs.
16
643978 = number of oz. in 40248 lbs. + 10 oz.
16
10303648 = number of drs. in 643978 oz.; that is in
17 tons 19 cwt. 1 qr. 12 lbs. 10 oz.

Obs.—The student should never use more than one line when multiplying by numbers between 12 and 20 (Art. **23**).

Ex. 3. Reduce 3 m. 7 fur. 20 po. 4 yds. to feet.

m. fur. po. yds.
3 7 20 4
8
31 = number of fur. in 3 m. + 7 fur.
40
1260 = number of po. in 31 fur. + 20 po.
$5\frac{1}{2}$
6304
630
6934 = number of yds. in 1260 po. + 4 yds.
3
20802 = number of ft. in 6934 yds.,
that is in 3 m. 7 fur. 20 po. 4 yds.

Obs.—To reduce poles to yards we have to multiply by $5\frac{1}{2}$; this is done by multiplying by 5 and adding to the product the number obtained by dividing the number of poles by 2; the sum will be the number of yards required. If the number of poles be odd we shall have, after dividing by 2, a remainder 1, and the sum will now be the number of yards + $\frac{1}{2}$ yard. This $\frac{1}{2}$ yard should be written as 1 ft. 6 in., which must be added in in their proper places together with the other feet and inches.

Ex. 4. Reduce 5 fur. 17 po. 3 yds. 1 ft. to inches.

fur. po. yds. ft.
5 17 3 1
40
217 = number of po. in 5 fur. + 17 po.
$5\frac{1}{2}$
1088
$108\frac{1}{2}$
1196 1 ft. 6 in. = number of yds., ft., and in. in
3 217 po. + 3 yds.
3590 = number of ft. in 1196 yds. + 2 ft.
12
43086 = number of inches in 3590 ft. + 6 in., that is in
5 fur. 17 po. 3 yds. 1 ft.

Ex. 5. Reduce 7 A. 3 R. 29 P. 17 S. yds. 5 S. ft. to square inches.

A. R. P. S. yds. S. ft.
7 3 29 17 5
4
31 = number of R. in 7 A. + 3 R.
40
1269 = number of P. in 31 R. + 29 P.
$30\frac{1}{4}$
38087
$317\frac{1}{4}$
38404 2 S. ft. 36 S. in. = number of S. yds., ft. and in.
9 in 1269 P. + 17 S. yds.
345643 = number of S. ft. in 38404 S. yds. + 7 S. ft.
144
1382608
4839002
49772628 = number of S. in. in 345643 S. ft. + 36 in.;
that is in 7 A. 3 R. 29 P. 17 S. yds. 5 S. ft.

Obs. 2.—To reduce perches to S. yards we have to multiply by $30\frac{1}{4}$. This is done by multiplying by 30 and adding to the product the number obtained by dividing the number of perches by 4; the sum will be the number of S. yards required. If the number of perches be not exactly divisible by 4 we shall have a remainder 1, 2, or 3, and the sum will be the number of s. yds. + $\frac{1}{4}$ s. yd., or $\frac{1}{2}$ s. yd., or $\frac{3}{4}$ s. yd., as the case may be. This should be expressed as feet and inches, which must be added in in their proper places.

87. *To reduce a quantity to a higher denomination.*

RULE.—Divide the proposed quantity by the number of units which make one of the next higher denomination, the remainder (if any) will be of the same denomination; as the dividend; and proceed in the same manner until the required denomination is arrived at.

Ex. 1. Reduce 257793 farthings to pounds.

$$
\begin{array}{ll}
\quad f. & \\
4)\underline{257793} & \\
12)\underline{64448}\ .\ 1 & = \text{number of pence and farthings in } 257793f. \\
2_\backslash 0)\underline{537_\backslash 0}\ .\ 8 & = \text{number of shillings and pence in } 64448d. \\
\quad \underline{268\ \ 10} & = \text{number of pounds and shillings in } 5370s.
\end{array}
$$

$\therefore$ 257793*f.* = £268 10*s.* $8\frac{1}{4}$*d.*

Ex. 2. Reduce 207899 ounces to tons.

$$
\begin{array}{r}
\text{oz.}\qquad\quad \\
16\left\{\begin{array}{r} 2)\dot{2}07899 \\ 8)\underline{103949}\ .\ 1 \end{array}\right. \\
28\left\{\begin{array}{r} 4)\underline{12993}\ .\ 5 \\ 7)\underline{3248}\ .\ 1 \end{array}\right. \\
4)\underline{464}\ .\ 0 \\
2_\backslash 0)\underline{11_\backslash 6} \\
\underline{5\ \ 16}
\end{array}
\quad
\begin{array}{l}
\\ \\
\left.\begin{array}{l} 1 \\ 5 \end{array}\right\}11\text{ oz.} \\
\left.\begin{array}{l} 1 \\ 0 \end{array}\right\}1\text{ lb.} \\
\\ \\
\end{array}
$$

$\therefore$ 207899 oz. = 5 tons 16 cwt. 1 lb. 11 oz.

Obs. 1.—When we divide by the two factors of a composite number separately, to get the complete remainder we must multiply the first divisor into the second remainder and add to the product the first remainder. Thus in Ex. 2 we have $2 \times 5 + 1 = 11$ and $0 \times 4 + 1 = 1$. The reason for this will be seen at once if we write the remainders as they arise in the form of fractions; thus, dividing by 2 in the above example, we get $\frac{1}{2}$, and in the second division there is a remainder 5, we have therefore to divide $5\frac{1}{2}$ by 8, but $5\frac{1}{2} \div 8 = \frac{11}{16}$ (Art. 44). The remainder, therefore, after dividing by 16, is 11.

If the divisor consists of three factors we must form the remainder corresponding to the two first, as above, then multiply the product of the two first factors into the third remainder and add to the product the remainder just formed corresponding to the two first factors.

Ex. 3. Reduce 7011975 feet to inches.

```
     3)7011975
     ─────────
       2337325
             2
   ───────────
   11)4674650
   ───────────
   4,0)42496,8 . 2 half yds. = 1 yd.
      ────────
     8)10624 . 8
     ────────
        1328
      ──────
```

∴ 7011975 ft. = 1328 m. 8 po. 1 yd.

Obs. 2.—To reduce yards to poles: since 5½ yards or 11 half yards make a pole, we first reduce the yards to half yards by multiplying by 2 and then divide by 11. As the remainder is always in the same denomination as the dividend, if there be any remainder in such a case it will denote not yards, but half yards. These should be expressed as yards, feet, &c., and added to the final result.

Again, to reduce square yards to perches: since 30¼ yards or 121 quarter yards make a perch, we first reduce the yards to quarter yards by multiplying by 4 and then divide by 121. If there be any remainder in such a case it will denote not yards, but quarter yards. These should be expressed as yards, feet, &c., and added to the final result.

Ex. 4. Reduce 527523 yards to furlongs.

```
        yds.
      527523
           2
    ────────
 11)1055046
    ────────
   4,0)9591,3 . 3 half yds. = 1 yd. 1 ft. 6 in.
    ─────────
     2397 . 33
    ──────
```

∴ 527523 yds. = 2397 fur. 33 po. 1 yd. 1 ft. 6 in.

Ex. 5. Reduce 15444933 square inches to acres.

s. in.
144 { 12)15444933
      12)1287077 . 9 } 69
       9)107256 . 5 }
         11917 . 3
             4
121 { 11)47668
      11)4333 . 5 } 115 qr. yds = 28 s. yds. 6 s. ft. 108 s. in.
   4,0)39,3 . 10 }
       4)9 . 33
         2 . 1

	A.	R.	P.	s. yds.	s. ft.	s. in.
	2	1	33	28	3	69
					6	108
∴ 15444933 s. in. =	2	1	33	29	1	33

Ex. 6. Reduce 75270787 cubic inches to cubic yards.

c. in.
1728 { 144 { 12)75270787
             12)6272565 . 7 } 115 } 835 c. in.
             12)522713 . 9 }
27 { 3)43559 . 5
     9)14519 . 2 } 8 c. ft.
       1613 . 2 }

∴ 75270787 c. in. = 1613 c. yds. 8 c. ft. 835 c. in.

COMPOUND ADDITION.

88. Rule.—Set the quantities to be added under one another, so that terms of the same kind may be in the same vertical column. Add together the numbers of the lowest denomination, and divide the sum by the number of such units as make one of the next higher denomination. Set the remainder (if any) under the column just added, add the quotient to the next column, and proceed as before.

Ex. 1. Add together £27 13*s.* 7½*d.*, £3 9*s.* 11*d.*, £340 12*s.* 2¼*d.*, £51 4*s.* 6¼*d.*, £101 13*s.* 9¾*d.*

£	*s.*	*d.*
27	13	7½
3	9	11
340	12	2¼
51	4	6¼
101	13	9¾
524	14	0¾

Adding up the first column we get 7 farthings, which, divided by 4 (because 4*f.* make 1*d.*), gives 1 and 3 over; we therefore set 3*f.* under the farthings column and carry 1 to the pence, which will make the sum of that column 36; this, divided by 12 (because 12*d.* make 1*s.*) gives 3 and nothing over; we therefore set 0 under the pence column and carry 3 to the shillings, which will make the sum of that column 54; this, divided by 20 (because 20*s.* make £1), gives 2 and 14 over; we therefore set 14 under the shillings column and carry 2 to the pounds, which will make the sum of the pounds 524.

Ex. 2. Add together 7 fur. 5 po. 3 yds., 1 fur. 37 po. 1 yd., 24 po. 4 yds., 5 fur. 3 po. 5 yds., 2 fur. 19 po. 3 yds.

fur.	po.	yds.
7	5	3
1	37	1
	24	4
5	3	5
2	19	3
17	10	5

Here, dividing 16, the sum of the yards, by 5½, we get 2 and 5 over; we therefore set 5 under the yards column and carry 2 to the poles, which will make the sum of that column 90; this divided by 40 gives 2 and 10 over; we therefore set 10 under the poles column and carry 2 to the furlongs, which will make the sum of that column 17.

COMPOUND SUBTRACTION.

89. Rule.—Set the quantity to be subtracted under the one from which it is to be taken, so that terms of the same kind may be in the same vertical column.

Subtract, if possible, the number of the lowest denomination in the lower line from the corresponding number in the upper line and set the difference beneath: proceed in the same manner with the next higher denomination, and so on throughout.

But if at any step the number in the lower line be greater than the corresponding number in the upper line, add to the number in the upper line as many units of that denomination as make one of the next higher denomination. From the number so formed subtract the number in the lower line and set the difference beneath, but before proceeding to subtract the next denomination, add 1 to that denomination in the lower line to compensate for the unit added to the upper line; then proceed as before.

The principle of this process of 'borrowing and carrying,' as it is usually called, consists in the fact that the difference between two numbers will not be affected if we add any number to one of them, provided we add the *same* number to the other.

Ex. 1. Subtract £5 14*s*. 6¾*d*. from £17 11*s*. 7½*d*.

£	*s.*	*d.*
17	11	7½
5	14	6¾
11	17	0¾

Since we cannot take 3*f*. from 2*f*. we add 4*f*. to the upper line, making in all 6*f*. Now 3*f*. from 6*f*. leave 3*f*.; we then set 3*f*. under the farthings, and having added 4*f*. or 1*d*. to the upper line we must also add 1*d*. to the lower line, making 7*d*. from 7*d*., which leaves 0. Again, since we cannot take 14*s*. from 11*s*., we add 20*s*. to the upper line, making in all 31*s*. Now 14*s*. from 31*s*. leave 17*s*.; we therefore set 17*s*. under the shillings, and having added 20*s*. or £1 to the upper line, we must also add £1 to the lower line, making £6 from £17, which leaves £11.

Ex. 2. Subtract 17 m. 2 fur. 20 po. 4 yds. from 25 m. 5 fur. 13 po. 3 yds.

m.	fur.	po.	yds.
25	5	13	3
17	2	20	4
8	2	32	4½

Since we cannot take 4 yards from 3 yards we add 5½ yards to the upper line, making in all 8½ yards. Now 4 yds. from 8½ yds. leave 4½ yds.; we therefore set 4½ yds. under the yards, and having added 5½ yards or 1 pole to the upper line, we must also add 1 po. to the lower line, making 21 po. from 13 po. We must now add 40 po. to the upper line, making in all 53 po., and proceed as before.

COMPOUND MULTIPLICATION.

90. Rule.—Place the multiplier under the lowest denomination of the multiplicand. Multiply the number of this lowest denomination by the multiplier, divide the product by the number of such units as make one of the next higher denomination, and set the remainder (if any) under this lowest denomination, add the quotient to the product obtained by multiplying the next higher denomination, and proceed in the same manner through all the denominations.

91. When the multiplier is a composite number greater than 12, it will generally be better to multiply separately by the factors of which it is composed.

If it be not a composite number we may multiply separately by the factors which make up the composite number *nearest* to it, and add to, or subtract from, the product, the product of the multiplicand multiplied by the difference between the composite number and the multiplier, according as the multiplier is greater or less than the composite number.

If the multiplier be very large, it will be better to reduce the multiplicand to its lowest denomination (Art. **86**), then multiply the result by the multiplier, and reduce the product back again.

Ex. 1. Multiply £10 17*s*. 3¼*d*. by 9.

£	*s*.	*d*.
10	17	3¼
		9
97	15	5¼

Here 1*f*. × 9 = 9*f*. = 2¼*d*., we therefore set 1*f*. under the farthings and carry 2*d*.; 3*d*. × 9 = 27*d*. which with the 2*d*. carried is 29*d*. = 2*s*. 5*d*.; we therefore set 5*d*. under the pence and carry 2*s*; 17*s*. × 9 = 153*s*., which with the 2*s*. carried is 155*s*. = £7 15*s*.; we therefore set down 15*s*. under the shillings and carry £7; £10 × 9 = £90, which with the £7 carried is £97, and ∴ £10 17*s*. 3¼*d*. × 9 = £97 15*s*. 3¼*d*.

Ex. 2. Multiply £14 3*s*. 7½*d*. by 36.

£	*s*.	*d*.	
14	3	7½	
		9	
127	12	7½	= £14 3*s*. 7½*d*. × 9
		4	
510	10	6	= £14 3*s*. 7½*d*. × 36.

Ex. 3. Multiply 4 fur. 12 po. 2 yds. by 67.

$$\begin{array}{rrl} \text{fur.} & \text{po.} & \text{yds.} \\ 4 & 12 & 2 \times 3 \\ & & 8 \\ \hline 34 & 18 & 5 = 4\text{ fur. }12\text{ po. }2\text{ yds.} \times 8 \\ & & 8 \\ \hline 275 & 31 & 1\frac{1}{2} = 4\text{ fur. }12\text{ po. }2\text{ yds.} \times 64 \\ 12 & 37 & 0\frac{1}{2} = 4\text{ fur. }12\text{ po. }2\text{ yds.} \times 3 \\ \hline 288 & 28 & 2 = 4\text{ fur. }12\text{ po. }2\text{ yds.} \times 67 \\ \hline \end{array}$$

$$\begin{aligned} 67 &= 64 + 3 \\ &= 8 \times 8 + 3 \end{aligned}$$

92. To multiply by 69 we should multiply by 10 and by 7, and subtract the first line from the product. To multiply by 157, we might multiply by 12 and by 12, and then add to the product the second line and the first, since $157 = 144 + 12 + 1 = (12 \times 12) + 12 + 1$.

So, to multiply by 173, we might multiply by 10 and by 10 and then add to the product 7 times the second line and 3 times the first; since $173 = 100 + 70 + 3 = (10 \times 10) + (10 \times 7) + 3$.

A little practice will enable the student to see at once the most convenient parts into which such factors may be broken up.

COMPOUND DIVISION.

93. Rule.—Place the divisor to the left of the dividend; then, if the divisor be not greater than 12, divide the highest denomination in the dividend by it, setting the quotient beneath the corresponding term in the dividend. Multiply the remainder (if any) by the number of units of the next lower denomination which make one of the preceding, add to the product the number of that denomination in the dividend, and divide as before; and so on through the several denominations.

If the divisor be greater than 12, and a composite number, divide by its several factors successively; but if the divisor be greater than 12 and not a composite number, place to the right of the dividend the part of the quotient obtained by dividing its first term. Multiply the remainder and add in the number of the next lower denomination, as before, but placing the successive terms of the quotient in their proper order to the right of the dividend.

Ex. 1. Divide £78 14*s.* 2*d.* by 8.

$$\begin{array}{rrr} \pounds & s. & d. \\ 8)78 & 14 & 2 \\ \hline 9 & 16 & 9\frac{1}{4} \\ \hline \end{array}$$

Here £78÷8 gives 9 and 6 remainder; 6 × 20 = 120, which with 14*s.* make 134*s.*; 134*s.*÷8 gives 16 and 6 remainder; 6*s.* × 12 = 72*d.*, which with 2*d.* make 74*d.*; 74÷8 gives 9 and 2 remainder; 2*d.* × 4 = 8*f.* and 8÷8 gives 1. ∴ £78 14*s.* 2*d.*÷8 = £9 16*s.* 9¼*d.*

Ex. 2. Divide £1008 15*s.* 8¼*d.* by 21.

$$\begin{array}{rrr} \pounds & s. & d. \\ 3)1008 & 15 & 8\frac{1}{4} \\ \hline 7)336 & 5 & 2\frac{3}{4} \\ \hline 48 & 0 & 8\frac{27}{28} \end{array}$$

Here 21 = 3 × 7; we therefore divide by 3 and 7 successively. In dividing by 7 we find that the last number of farthings is not exactly divisible by 7. From the pence we have 6 remainder, and have therefore to divide 6¾ by 7, but 6¾÷7 = $\frac{27}{28}$ (Art. **44**), which is therefore the required fraction of a penny.

Ex. 3. Divide £137 17*s.* 7½*d.* by 26.

$$\begin{array}{rrrrrr} \pounds & s. & d. & \pounds & s. & d. \\ 26)137 & 17 & 7\frac{1}{2}(5 & 6 & 0\frac{3}{4} \\ 130 \\ \hline 7 \\ 20 \\ \hline 157 \\ 156 \\ \hline 1 \\ 12 \\ \hline 19 \\ 4 \\ \hline 78 \\ 78 \\ \hline .. \end{array}$$

∴ £137 17*s.* 7½*d.*÷26 = £5 6*s.* 0¾*d.*

94. When a concrete quantity is divided by an abstract number the quotient is a concrete quantity of the same kind, and, if added to itself the number of times expressed by the divisor will produce the dividend; but when one concrete quantity is to be divided by another, we must first reduce them both to the same denomination, and then divide as if they were abstract numbers. The quotient will now be an *abstract number* expressing the number of times the *divisor* must be added to itself to produce the dividend.

Ex. 1. Divide £97 15*s.* $9\frac{3}{4}$*d.* by £10 17*s.* $3\frac{3}{4}$*d.*

£97 15*s.* $9\frac{3}{4}$*d.* = 93879*f.*; £10 17*s.* $3\frac{3}{4}$*d.* = 10431*f.*

$$\begin{array}{r} 10431)93879(9 \\ \underline{93879} \\ \cdots\cdot \end{array}$$

Hence £10 17*s.* $3\frac{3}{4}$*d.* multiplied by 9 or added to itself 9 times will produce £97 15*s.* $9\frac{3}{4}$*d.*

REDUCTION OF FRACTIONS.

95. ***To find the value of a fractional part of a quantity.***

Rule.—Multiply the quantity by the numerator of the fraction, and divide the product by the denominator.

If it be a mixed number, multiply the quantity by the whole number, and add to the product the value of the fractional part found as above.

Ex. 1. Find the value of $\frac{5}{6}$ of £2 13*s.*

$$\begin{array}{rrrr} & £ & s. & \\ & 2 & 13 & \\ & & 5 & \\ \hline 6) & 13 & 5 & \\ \hline & 2 & 4 & 2 \\ \hline \end{array}$$

∴ $\frac{5}{6}$ of £2 13*s.* = £2 4*s.* 2*d.*

Ex. 2. Find the value of $3\frac{5}{9}$ of a guinea.

$$3\tfrac{5}{9}\text{ gui.}=3\text{ gui.}+\tfrac{5}{9}\text{ gui.};\ \tfrac{5}{9}\text{ gui.}=\frac{5\times\overset{7}{\cancel{21}}}{\underset{3}{\cancel{9}}}s.=\tfrac{35}{3}s.=11\tfrac{2}{3}s.;\ \tfrac{2}{3}s.=\frac{2\times\overset{4}{\cancel{12}}}{\cancel{3}}d.=8d.$$

∴ $\frac{5}{9}$ gui. = 11*s.* 8*d.*

$$\begin{array}{rrrr} & £ & s. & d. \\ & 3 & 3 & 0 \\ & & 11 & 8 \\ \hline \therefore 3\tfrac{5}{9}\text{ gui.}= & 3 & 14 & 8 \end{array}$$

Ex. 3. Find the value of $7\frac{5}{12}$ of 5 cwt. 2 qrs. 16 lbs.

$$\begin{array}{rrrrr}
\text{cwt.} & \text{qrs.} & \text{lbs.} & & \\
5 & 2 & 16 & & \\
 & & 7 & & \\
\hline
39 & 2 & 0 & & \\
2 & 1 & 11 & 5 & 5\frac{1}{3} \\
\hline
41 & 3 & 11 & 5 & 5\frac{1}{3} \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rrrrr}
\text{cwt.} & \text{qrs.} & \text{lbs.} & & \\
5 & 2 & 16 & & \\
 & & 5 & & \\
\hline
12)28 & 0 & 24 & & \\
\hline
2 & 1 & 11 & 5 & 5\frac{1}{3} \\
\hline
\end{array}$$

∴ $\frac{5}{12}$ of 5 cwt. 2 qrs. 16 lbs. = 2 cwt. 1 qr. 11 lbs. 5 oz. $5\frac{1}{3}$ drs.
and $7\frac{5}{12}$ of 5 cwt. 2 qrs. 16 lbs. = 44 cwt. 0 qrs. 22 lbs. 10 oz. $10\frac{2}{3}$ drs.

Ex. 4. Find the value of $\frac{3}{4}$ of £17 + $\frac{7}{18}$ of a moidore − $1\frac{7}{9}$ of £2 5*s*.

$$\begin{array}{r}
£ \\
17 \\
3 \\
\hline
4)51 \\
\hline
12\ \ 15s. \\
\hline
\end{array}
\qquad
\frac{7}{18} \text{ of } 27s. = \frac{7 \times \overset{3}{\cancel{27}}}{\underset{2}{\cancel{18}}}s. = \frac{21}{2}s. = 10s.\ 6d.$$

$$1\frac{7}{9} \text{ of } £2\ 5s. = £2\ 5s. + \frac{7}{9} \text{ of } £2\ 5s.$$

$$\begin{array}{rr}
£ & s. \\
2 & 5 \\
 & 7 \\
\hline
9)15 & 15 \\
\hline
1 & 15 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rr}
£ & s. \\
2 & 5 \\
1 & 15 \\
\hline
4 & 0 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rrr}
£ & s. & d. \\
12 & 15 & 0 \\
 & 10 & 6 \\
\hline
13 & 5 & 6 \\
4 & 0 & 0 \\
\hline
9 & 5 & 6 \\
\hline
\end{array}$$

Ans. £9 5*s*. 6*d*.

96. *To express one quantity as the fraction of another.*

Rule.—Reduce both quantities to the same denomination.

The fraction formed by taking the former for the numerator and the latter for the denominator will be the fraction required.

Ex. 1. Reduce 3*s*. 4*d*. to the fraction of 17*s*. 8*d*.

3*s*. 4*d* = 40*d*.
17*s*. 8*d*. = 212*d*. ∴ fraction required is $\frac{40}{212} = \frac{10}{53}$ *Ans.*

Ex. 2. Reduce 1 cwt. 3 qrs. 12 lbs. to the fraction of 3 tons 12 cwt.

1 cwt. 3 qrs. 12 lbs. = 208 lbs.
3 tons 12 cwt. = 8064 lbs. ∴ the fraction required is $\frac{208}{8064} = \frac{13}{502}$ *Ans.*

97. When it is required to express a quantity as the fraction of another of only one denomination, we may avoid the labour of reduction by proceeding as in the following example.

Ex. 3. Reduce £2 13*s.* $4\frac{1}{2}$*d.* to the fraction of £4.

$4\frac{1}{2}d. = \frac{9}{2}d. = \frac{9}{2\times12}s. = \frac{3}{8}s.$; $13\frac{3}{8}s. = \frac{107}{8}s. = £\frac{107}{8\times20} = £\frac{107}{160}$; $£2\frac{107}{160} = £\frac{427}{160}$

$= \frac{427}{160\times4}$ of $£4 = \frac{427}{640}$, which is therefore the fraction required.

Ex. 4. What fraction of a ton is 6 cwt. 1 qr. 8 lbs. 12 oz.?

This question is the same as Reduce 6 cwt. 1 qr. 8 lbs. 12 oz. to the fraction of a ton.

12 oz. $= \frac{12}{16}$ lb. $= \frac{3}{4}$ lb.; $8\frac{3}{4}$ lb. $= \frac{35}{4}$ lb. $= \frac{\overset{5}{\cancel{35}}}{4\times\underset{4}{\cancel{28}}}$ qr. $= \frac{5}{16}$ qr.; $1\frac{5}{16}$ qr. $= \frac{21}{16}$ qr.

$= \frac{21}{16\times4}$ cwt. $= \frac{21}{64}$ cwt. $6\frac{21}{64}$ cwt. $= \frac{405}{64}$ cwt. $= \frac{\overset{81}{\cancel{405}}}{64\times\underset{4}{\cancel{20}}}$ ton $= \frac{81}{256}$, which

is therefore the fraction required.

Ex. 5. Express $\frac{3}{5}$ of £3 7*s.* 6*d.* + $\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ gui. as the fraction of £3.

$\frac{3}{5}$ of £3 7*s.* 6*d.* $= \frac{3}{5}$ of $£3\frac{3}{8} = \frac{3}{5}$ of $£\frac{27}{8} = £\frac{81}{40} = \frac{81\times\cancel{20}}{\underset{2}{\cancel{40}}}s. = \frac{81}{2}s.$;

$\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ gui. $= \frac{2}{3}$ of $\frac{1}{4}$ of $\frac{9}{2}$ gui. $= \frac{\cancel{2}}{\cancel{3}}$ of $\frac{1}{4}$ of $\frac{\overset{3}{\cancel{9}}\times21}{\cancel{2}}s. = \frac{63}{4}s.$;

$\frac{81}{2}s. + \frac{63}{4}s. = \frac{162+63}{4}s. = \frac{225}{4}s. = £\frac{\overset{45}{\cancel{225}}}{4\times\underset{4}{\cancel{20}}} = £\frac{45}{16}$; and $\frac{45}{16}$ of $£1 = \frac{\overset{15}{\cancel{45}}}{16\times\cancel{3}}$

of $£3 = \frac{15}{16}$, which is therefore the fraction required.

REDUCTION OF DECIMALS.

98. *To find the value of a decimal of any quantity.*

RULE.—Multiply the decimal by the number of units of the next lower denomination which make 1 of the given quantity. The integral part (if any) of the product will be the whole number of that lower denomination, and the value of the decimal part may be found in the same manner in terms of the next lower denomination, and so on.

Obs.—If the given quantity be expressed in more than one denomination, it must be reduced to the lowest denomination mentioned before applying the rule.

Ex. 1. Find the value of 3·8725 of £1.

$$\begin{array}{r} \pounds \\ 3{\cdot}8725 \\ 20 \\ \hline 17{\cdot}4500 \\ 12 \\ \hline 5{\cdot}40 \\ \hline \end{array}$$

$\cdot 4d. = \frac{4}{10}d. = \frac{2}{5}d.$ ∴ £3·8725 = £3 17*s.* 5⅖*d.*

Ex. 2. Find the value of 7·2375 of an acre.

$$\begin{array}{r} \text{A.} \\ 7{\cdot}2375 \\ 4 \\ \hline {\cdot}9500 \\ 40 \\ \hline 38{\cdot}00 \\ \hline \end{array}$$

∴ 7·2375 A. = 7 A. 38 P.

Ex. 3. Find the value of ·052 of £2 15*s.*

£2 15*s.* = 55*s.*

$$\begin{array}{r} s. \\ 55 \\ {\cdot}052 \\ \hline 110 \\ 275 \\ \hline 2{\cdot}860 \\ 12 \\ \hline 10{\cdot}32 \\ \hline \end{array}$$

$\cdot 32d. = \frac{32}{100}d. = \frac{8}{25}d.$ ∴ ·052 of £2 15*s.* = 2*s.* $10\frac{8}{25}d.$

Ex. 4. Find the value of $\cdot 4\dot{9}$ of £3 15*s.* 6*d.*

(Art. 65.) $\cdot 4\dot{9} = \dfrac{49-4}{90} = \frac{45}{90} = \frac{1}{2}$.

½ of £3 15*s.* 6*d.* = £1 17*s.* 9*d.* ∴ $\cdot 4\dot{9}$ of £3 15*s.* 6*d.* = £1 17*s.* 9*d.*

Ex. 5. Find the value of $2{\cdot}7\dot{4}5\dot{2}$ of £4 3*s.* 3*d.*

$2{\cdot}7\dot{4}5\dot{2} = 2\dfrac{7452-7}{9990} = 2\frac{7445}{9990}$;

$2\frac{7445}{9990}$ of £4 3*s.* 3*d.* = £8 6*s.* 6*d.* + $\frac{7445}{9990}$ of 999*d.*;

$\frac{7445}{9990}$ of 999*d.* = $\frac{7445}{10}d. = 744\frac{1}{2}d.$ = £3 2*s.* 0½*d.*

	£	*s.*	*d.*
	8	6	6
	3	2	0½
Ans.	£11	8	6½

99. *To reduce a quantity to the decimal of another given quantity.*

RULE.—Express the first quantity as the fraction of the second (Art. **96**), and reduce this fraction to a decimal (Art. **59**); this will be the decimal required.

Ex. 1. Reduce 1*s.* 9*d.* to the decimal of 2*s.* 1*d.*
1*s.* 9*d.* = 21*d.*
2*s.* 1*d.* = 25*d.* and $\frac{21}{25} = \cdot84$, the decimal required.

Ex. 2. Reduce 1 lb. 6 oz. 8 dwts. to the decimal of 2 lbs. 15 dwts.
1 lb. 6 oz. 8 dwts. = 368 dwts.
2 lbs. 0 oz. 15 dwts. = 495 dwts. and $\frac{368}{495} = \cdot7\dot{4}\dot{3}$ (Art. **62**, Ex. 1).

100. When the quantity to the decimal of which the other is to be reduced is expressed in one denomination only, the following rule may be used.

RULE.—Divide the lowest denomination of the proposed quantity by the number of units of that denomination which make one of the next higher. Prefix to the decimal thus obtained the corresponding term (if any) of this higher denomination in the proposed quantity, and divide the number so formed by the number of units of this denomination which make one of the next higher, and so on through all the denominations.

Ex. 1. Reduce 17*s.* 10½*d.* to the decimal of £1.

$$\begin{array}{r} 4\overline{)2\cdot0} \\ 12\overline{)10\cdot5} \\ 20\overline{)17\cdot875} \\ \cdot89375 \end{array}$$

∴ ·89375 is the decimal required.

Ex. 2. Reduce 5 cwt. 3 qrs. 12 lbs. 4 oz. to the decimal of 8 cwt.

$$\begin{array}{r} 16\overline{)4\cdot00} \\ 28\begin{cases} 4\overline{)12\cdot25} \\ 7\overline{)3\cdot0625} \end{cases} \\ 4\overline{)3\cdot4375} \\ 8\overline{)5\cdot859375} \\ \cdot732421875 \end{array}$$

∴ ·732421875 is the decimal required.

PRACTICE.

101. *Def.*—An *Aliquot Part* of any quantity is such a part as will make up the quantity when repeated some exact number of times, and may always be expressed as a fraction of the given quantity, with 1 for its numerator. Thus 1*d.*, $1\frac{1}{2}$*d.*, 2*d.*, 3*d.*, 4*d.*, 6*d.*, are all aliquot parts of a shilling, the corresponding fractions being $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$, respectively. So also 2 lbs., 4 lbs., 7 lbs., 14 lbs., are aliquot parts of a quarter, the corresponding fractions being $\frac{1}{14}$, $\frac{1}{7}$, $\frac{1}{4}$, and $\frac{1}{2}$ respectively.

102. *Practice* is a convenient method of finding the value of a given quantity when the value of a unit of any denomination in the quantity is known.

The value of a quantity is found by finding the sum of the values of the aliquot parts.

There are two cases.

Case I.—When the given quantity is expressed in the same denomination as the unit whose value is known.

Case II.—When the given quantity is not wholly expressed in the same denomination as the unit whose value is known.

103. The method of working examples in practice will be shown in the following examples.

Ex. 1. Find the cost of 822 articles at 16*s.* 6*d.* each.

10*s.* 0*d.*	$\frac{1}{2}$	822			
5*s.* 0*d.*	$\frac{1}{2}$	411		. . .	cost at 10*s.* 0*d.* each
1*s.* 0*d.*	$\frac{1}{5}$	205	10	. . .	„ 5*s.* 0*d.* „
6*d.*	$\frac{1}{2}$	41	2	. . .	„ 1*s.* 0*d.* „
		20	11	. . .	„ 6*d.* „
		£678	3	. . .	„ 16*s.* 6*d.* „

Since 10*s.* is the half of £1, we shall find how many pounds any number of articles are worth at 10*s.* each by dividing the number by 2. Again, since 5*s.* is the half of 10*s.*, we shall find the value at 5*s.* each by dividing the value at 10*s.* each by 2. So also, since 1*s.* is the $\frac{1}{5}$ of 5*s.*, we shall find the value at 1*s.* each by dividing the value at 5*s.* each by 5. And, lastly, since 6*d.* is the half of 1*s.*, we shall find the value at

6*d.* each by dividing the value at 1*s.* each by 2. But 10*s.* + 5*s.* + 1*s.* + 6*d.* = 16*s.* 6*d.*; therefore £678 3*s.*, the sum of these values, must be the value at 16*s.* 6*d.* each.

Ex. 2. Find the value of 20724 articles at £1 13*s.* 8½*d.* each.

		£	*s.*	*d.*			
10*s.* 0*d.*	½	20724	0	0	. .	value at £1	0*s.* 0*d.* each
2*s.* 6*d.*	$\frac{1}{10}$ ¼	10362	0	0	. .	„	10*s.* 0*d.* „
1*s.*		2590	10	0	. .	„	2*s.* 6*d.* „
2*d.*	⅙	1036	4	0	. .	„	1*s.* 0*d.* „
½*d.*	¼	172	14	0	. .	„	2*d.* „
		43	3	6	. .	„	½*d.* „
		£34928	11	6	. .	„	£1 13*s.* 8½*d.* „

Here the first line is the value at £1 each; then 10*s.* is the ½ of £1; 2*s.* 6*d.* is the ¼th of 10*s.*; 1*s.* is the $\frac{1}{10}$th of 10*s.*; and we must be careful here to take the $\frac{1}{10}$th, not of the last line, but *of the line expressing the value at* 10*s. each*; 2*d.* is the ⅙th of 1*s.*; and ½ is the ¼th of 2*d.* Now the sum of these aliquot parts is £1 13*s.* 8½*d.*; and ∴ £34928 11*s.* 6*d.*, the sum of the values, is the value required.

Ex. 3. Find the value of 10542 articles at £3 17*s.* 11¾*d.* each.

10*s.* 0*d.*	½	10542					
		3					
		31626			. .	value at £3	0*s.* 0*d.* each
5*s.* 0*d.*	½	5271			. .	„	10*s.* 0*d.* „
2*s.* 6*d.*	½	2635	10	0	. .	„	5*s.* 0*d.* „
5*d.*	⅙	1317	15	0	. .	„	2*s.* 6*d.* „
½*d.*	$\frac{1}{10}$	219	12	6	. .	„	5*d.* „
¼*d.*	½	21	19	3	. .	„	½*d.* „
		10	19	7½	. .	„	¼*d.* „
		£41102	16	4½			

Here we first multiply by 3 to find the value at £3 each, and then take aliquot parts for the 17*s.* 11¾*d.*

104. CASE II.

Ex. 1. Find the value of 5 cwt. 2 qrs. 21 lbs. at £4 10*s.* per cwt.

		£	*s.*	*d.*			
2 qrs. 0 lbs.	½	4	10		. .	value of 1 cwt.	
			5				
		22	10		. .	„	5 cwt.
14 lbs.	¼	2	5		. .	„	2 qrs.
7 lbs.	½		11	3	. .	„	14 lbs.
			5	7½	. .	„	7 lbs.
		£25	11	10½	. .	„	5 cwt. 2 qrs. 21 lbs.

Here we multiply the value of 1 cwt. by 5, to get the value of 5 cwt., and then take parts as follows: 2 qrs. is the $\frac{1}{2}$ of 1 cwt.; 14 lbs. is the $\frac{1}{4}$th of 2 qrs.; and 7 lbs. is the $\frac{1}{2}$ of 14 lbs.

Ex. 2. Find the value of 9 cwt. 1 qr. 16 lbs. 13 oz. at £12 13*s.* 4*d.* per cwt.

		£	*s.*	*d.*		
1 qr.	$\frac{1}{4}$	12	13	4	. . value of 1 cwt.	
				9		
		114	0	0	. . „	9 cwt.
4 lbs.	$\frac{1}{2}$	3	3	4	. . „	1 qr.
2 lbs.	$\frac{1}{7}$	1	11	8	. . „	14 lbs.
8 oz.	$\frac{1}{4}$		4	$6\frac{2}{7}$	. . „	2 lbs.
4 oz.	$\frac{1}{2}$		1	$1\frac{4}{7}$	. . „	8 oz.
1 oz.	$\frac{1}{4}$			$6\frac{11}{14}$	. . „	4 oz.
				$1\frac{39}{56}$	. . „	1 oz.
		£119	1	$4\frac{19}{56}$	. . „	9 cwt. 1 qr. 16 lbs. 13 oz.

The parts taken here are 1 qr. the $\frac{1}{4}$th of 1 cwt.; 14 lbs. the $\frac{1}{2}$ of 1 qr.; 2 lbs. the $\frac{1}{7}$th of 14 lbs.; 8 oz. the $\frac{1}{4}$th of 2 lbs.; 4 oz. the $\frac{1}{2}$ of 8 oz.; and 1 oz. the $\frac{1}{4}$th of 4 oz.

Ex. 3. What is the rent of 23 a. 3 r. 5 p. at £2 12*s.* 6*d.* per acre?

		£	*s.*	*d.*		
2 r.	$\frac{1}{2}$	2	12	6	. . rent of 1 acre	
				12		
		31	10	0		
				2		
		63	0	0		
		2	12	6		
		60	7	6	. . „	23 acres
1 r.	$\frac{1}{2}$	1	6	3	. . „	2 roods
5 p.	$\frac{1}{8}$		13	$1\frac{1}{2}$	. . „	1 rood
			1	$7\frac{11}{16}$	. . „	5 perches
		£62	8	$6\frac{3}{16}$	. . „	23 a. 3 r. 5 p.

Ex. 4. What will be the annual income from an estate worth £1384 16*s.* a year after deducting a land tax of 2*s.* 9$\frac{1}{2}$*d.* in the pound?

		£	*s.*	*d.*	
2*s.* 6*d.*	$\frac{1}{8}$	1384	16	0	
3*d.*	$\frac{1}{10}$	173	2	0	tax at 2*s.* 6*d.* in the pound
$\frac{1}{2}$*d.*	$\frac{1}{6}$	17	6	$2\frac{2}{5}$	„ 3*d.* „
		2	17	$8\frac{2}{5}$	„ $\frac{1}{2}$*d.* „
		193	5	$10\frac{4}{5}$	„ 2*s.* 9$\frac{1}{2}$*d.* „
		1384	16	0	
		193	5	$10\frac{4}{5}$	
		£1191	10	$1\frac{1}{5}$	is therefore the annual income.

PROPORTION.

105. *Def.*—When the first of four quantities is the sa multiple *or* fraction of the second which the third is of t fourth, the four are said to be proportional. Thus 3, 9, 18, are proportional, since each of the fractions $\frac{3}{9}$, $\frac{6}{18}$, equal to $\frac{1}{3}$. So, also, 24, 8, 3, 1, are proportional, sin $\frac{24}{8}$, $\frac{3}{1}$, are each equal to 3.

106. *Def.*—The *ratio* of two quantities is the relati which the former bears to the latter in respect of mag tude; the comparison being made by considering wh multiple or fraction the former is of the latter, and is e pressed by writing the two quantities with two d between them. Thus 48 : 64; 25 : 35; £2 16*s.* : £3 4 the corresponding fractions being $\frac{48}{64}=\frac{3}{4}$, $\frac{25}{35}=\frac{5}{7}$, $\frac{56}{64}=\frac{7}{8}$.

It is evident that the terms of which a ratio is compos must consist of quantities of the same kind.

107. It will be seen from the two last articles that wh two ratios are equal the four quantities are proportion Thus 20 : 25 and 36 : 45 form a proportion, since each the fractions $\frac{20}{25}$, $\frac{36}{45}$, is equal to $\frac{4}{5}$.

A proportion is expressed by writing four dots : : betwe the two ratios of which it is composed. Thus 3 : 9 : : 6 : 1 20 : 25 : : 36 : 45.

108. When four quantities are proportional, the produ of the first and last (which are called the *extremes*) is alwa equal to the product of the second and third (which a called the *means*). Thus 3 : 9 : : 6 : 18; $3\times18=54=9\times$ 24 : 8 : : 3 : 1; $24\times1=24=8\times3$; 20 : 25 : : 36 : 45, $20\times$ $=900=25\times36$.

109. It follows from Art. **108** that if we know thr terms of a proportion we can always find the fourth; f when any number is known to be the product of tw factors, one of which is given, the other factor may imm factor. Thus if we know the three first terms of a proportion, the fourth term $=\frac{\text{2nd term} \times \text{3rd term}}{\text{1st term}}$.

110. We see from Art. **102** that the first and second terms of a proportion *must* be of the same kind, and so also the third and fourth, but not necessarily the second and third.

It must be remembered also that a ratio *always* represents an *abstract number*, although its terms may consist of concrete numbers, for the division of one concrete number by another can only give an abstract number for the quotient (Art. **94**).

SINGLE RULE OF THREE.

111. In all questions to which this rule is applicable, it will be found that three quantities are given, two of which are of the same kind, and a third of the same kind as the one which is required to be found, and which must be such that the four quantities may be proportional. It is evident then, from Art. **110**, that the two given quantities of the same kind form the first and second terms of the proportion, the other given quantity the third, and the one required the fourth.

From Art. **109** we see that if the answer, that is the fourth term, will be greater than the third, that term must be multiplied by the *greater* of the other two terms, or, in other words, the greater of those terms will form the second term of the proportion; but if the answer will be less, then the lesser of those terms will form the second term of the proportion.

From these considerations we obtain the following

Rule.—Set the single term last, and the greater or lesser of the other two terms second, according as the answer required will be greater or less than the third term.

Reduce the first and second terms to the same denomination, and, if necessary, the third to the lowest denomination in it. Divide the product of the second and third terms by the first. The quotient will be the answer required in the same denomination as the third term was left in.

Ex. 1. If 5 bushels of wheat cost £2 3*s.* 9*d.*, how much must be paid for 7 bushels?

$$\begin{array}{ccccrrrrl}
\text{bus.} & & \text{bus.} & & & \pounds & s. & d. & \\
5 & : & 7 & :: & & 2 & 3 & 9 & \\
 & & & & & & & 7 & \\ \cline{6-8}
 & & & & 5) & 15 & 6 & 3 & \\ \cline{6-8}
 & & & & & 3 & 1 & 3 & \textit{Ans.} \\ \cline{6-8}
\end{array}$$

Here £2 3*s.* 9*d.* is the single term corresponding to the answer, which will evidently be greater than £2 3*s.* 9*d.*, since 7 bushels must cost more than 3 bushels; we therefore set £2 3*s.* 9*d.* in the third term, 7, the greater of the other two, in the second, and 5, the remaining one, in the first.

In this example no reduction is required.

Ex. 2. If a man earns £191 12*s.* 6*d.* in a year, how long will he be earning £31 10*s.*?

£191 12*s.* 6*d.* : £31 10*s.*::365.

Here 1 year or 365 days is the single term corresponding to the answer, which will evidently be less than a year; we therefore set 365 in the third term, £31 10*s.*, the lesser of the two others, in the second, and £191 12*s.* 6*d.*, the remaining one, in the first. We now reduce the first and second terms to sixpences, and find the answer to be 60 days.

Obs.—Since the product of the second and third terms is to be divided by the first, we should always, before multiplying them together, cancel out any factors which may be common to the first and second, or to the first and third, being very careful, however, never to cancel between the *second and third.* This will save much unnecessary labour, and, indeed, whenever the student has several numbers to multiply together he should always keep them in the form of factors as long as possible, and not perform the actual multiplication until obliged to do so.

Ex. 3. In what time will 3 men do a piece of work which 12 men can do in 25 days?

$$\cancel{3} : \underset{4}{\cancel{12}} :: 25. \quad 25 \times 4 = 100.$$

Ans. 100 days.

Ex. 4. If the carriage of 3 cwt. 14 lbs. for 97 miles cost £1 7*s.* 1*d.*, for how much ought 11 cwt. 3 qrs. to be carried the same distance?

The distance, 97 miles, being the same in both cases, may be left out of consideration, and we have

$$\begin{array}{r} \text{cwt. lbs.} \\ 3\ \ 14 \\ 112 \\ \hline \cancel{350} \\ \cancel{70} \\ \cancel{14} \\ \cancel{7} \end{array} \ : \ \begin{array}{r} \text{cwt. qrs.} \\ 11\ \ \ 3 \\ 4 \\ \hline 47 \\ 28 \\ \hline 376 \\ 94 \\ \hline \cancel{1316} \\ \cancel{658} \\ 94 \end{array} \ :: \ \begin{array}{r} £\ \ s.\ \ d. \\ 1\ \ 7\ \ 1 \\ 20 \\ \hline 27 \\ 12 \\ \hline \cancel{325} \\ \cancel{65} \\ 13 \end{array} \qquad \begin{array}{r} 94 \\ 13 \\ \hline 12)1222 \\ \hline 2,0)10,1-10 \\ \hline 5-1 \\ \hline \end{array} \ \text{(Art. 23.)}$$

Ans. £5 1*s.* 10*d.*

Ex. 5. A bankrupt owes £864 9*s.* 4*d.* and his assets are £540 5*s.* 10*d.*; how much will a creditor receive to whom he owes £40 6*s.* 8*d.*?

Here £864 9*s.* 4*d.* and £40 6*s.* 8*d.* are *debts*; but £540 5*s.* 10*d.* *property*; this is therefore the single term, and as the creditor will receive less than £40 6*s.* 8*d.*, we have

$$\begin{array}{r} £\ \ \ s.\ \ d. \\ 864\ \ 9\ \ 4 \\ 20 \\ \hline 17289 \\ 12 \\ \hline \cancel{207472} \\ \cancel{25934} \end{array} \ : \ \begin{array}{r} £\ \ \ s.\ \ d. \\ 40\ \ 6\ \ 8 \\ 20 \\ \hline 806 \\ 12 \\ \hline \cancel{9680} \\ 1210 \end{array} \ :: \ \begin{array}{r} £\ \ \ s.\ \ d. \\ 540\ \ 5\ \ 10 \\ 20 \\ \hline 10805 \\ 12 \\ \hline \cancel{129670} \\ 5 \end{array} \qquad \begin{array}{r} d. \\ 12)6050 \\ \hline 2,0)50,4-2 \\ \hline \end{array}$$

$$1210 \times 5 = 6050$$

Ans. £25 4*s.* 2*d.*

If we had wanted to find how much the creditors would receive in the pound, the £1 being a *debt* would have been the second term instead of the £40 6*s*. 8*d*., and the answer would then be 12*s*. 6*d*.

Obs.—Whenever the second term is pounds alone, the multiplication by 20 and 12 should never be actually performed, but only indicated by signs. Thus, in Ex. 5, if the second term had been £40 instead of £40 6*s*. 8*d*., the working, after reducing the first and third terms, would have stood thus—

$$\frac{\overset{5}{\cancel{40}} \times \cancel{20} \times \cancel{12} \times \overset{5}{\cancel{129670}}}{\underset{\cancel{25934}}{\cancel{207472}} \times \cancel{20} \times \cancel{12}} = £25.$$

The 20 and 12 cancelling out with the 20 and 12 in the denominator, and giving the answer at once in pounds.

Ex. 6. If $\frac{7}{18}$ of a share be worth £36 15*s*., what is the value of $\frac{5}{14}$ of the share?

$$\tfrac{7}{18} : \tfrac{5}{14} :: £36\ 15s. \qquad £36\ 15s. = £36\tfrac{3}{4} = £\tfrac{147}{4}\ ,$$

$$\frac{\overset{9}{\cancel{18}}}{\cancel{7}} \times \frac{5}{\underset{\cancel{2}}{\cancel{14}}} \times \frac{\overset{\overset{3}{\cancel{21}}}{\cancel{147}}}{4} = £\tfrac{135}{4} = £33\tfrac{3}{4} = £33\ 15s.$$

Whenever, as in this example, the first term is a fraction, we invert it, and then multiply the three terms so formed together (Art. 44).

112. There are numerous questions to which this rule is applicable, which may require some ingenuity on the part of the student to reduce them to a form in which the rule can be applied.

Ex. 1. A hare gets up 201 yards before a greyhound. Supposing the greyhound to run 9 yards while the hare runs 6 yards, how far will the hare run before she is caught?

The greyhound will gain 3 yards for every 6 yards the hare runs, and when he has gained 201 yards he will catch her. Hence 3 yards and 201 yards represent distances gained by the dog, and 6 yards a distance run by the hare. This is therefore the single term, and we have

$$\cancel{3} : 201 :: \underset{2}{\cancel{6}}. \qquad 201 \times 2 = 402.$$

The hare therefore runs 402 yards.

Ex. 2. After paying income tax at 15*d.* in the pound, a gentleman has £465 16*s.* 3*d.*; what is his gross annual income?

After paying the tax 18*s.* 9*d.* is left out of every pound; hence

$$
\begin{array}{rcrcr}
\begin{array}{r} s.\ \ d. \\ 18\ \ 9 \\ 4 \\ \hline \cancel{75} \\ 15 \end{array}
& : &
\begin{array}{r} £\ \ \ s.\ \ d. \\ 465\ \ 16\ \ 3 \\ 20 \\ \hline 9316 \\ 4 \\ \hline \cancel{37265} \end{array}
& :: &
\begin{array}{r} £ \\ 1 \\ \\ \\ \\ \\ \end{array}
\end{array}
$$

$$
15\left\{\begin{array}{r} 5\overline{)7453} \\ 3\overline{)1490} \\ 496 \end{array}\right.
\left.\begin{array}{l} \\ .\ 3 \\ .\ 2 \end{array}\right\}13
$$

$$
£\tfrac{13}{15}=\frac{13\times\overset{4}{\cancel{20}}}{\underset{3}{\cancel{15}}}s.=\tfrac{52}{3}s.=17\tfrac{1}{3}s.=17s.\ 4d.
$$

His gross income is therefore £496 17*s.* 4*d.*

Ex. 3. Find the time between 10 and 11 o'clock when the hour and minute hands of a watch are (1) exactly opposite, (2) coincident.

(1) At 10 o'clock the hour hand is 50 of the spaces marking minutes in advance of the minute hand, and when they are exactly opposite it be 30 in advance; we have therefore to find the time when the minute hand will have gained 20 such spaces. Now the minute hand moves 12 times as fast as the hour hand, i.e. moves over 1 of these spaces in a minute, whilst the hour hand moves over only $\frac{1}{12}$ of the same distance in a minute; the minute hand therefore gains $\frac{11}{12}$ of the space marking a minute in each minute of time; hence we have

$$\tfrac{11}{12}:20::1\text{ min. }\frac{12\times20}{11}=\tfrac{240}{11}=21\tfrac{9}{11}\text{ min.}$$

they will ∴ be exactly opposite at $21\frac{9}{11}$ min. past 10 o'clock.

(2) They will be coincident when the minute hand has gained 50 such spaces; hence we have

$$\tfrac{11}{12}:50::1\text{ min. }\frac{12\times50}{11}=\tfrac{600}{11}=54\tfrac{6}{11}\text{ min.}$$

They will ∴ be coincident at $54\frac{6}{11}$ min. past 10 o'clock.

DOUBLE RULE OF THREE.

113. This is a shorter method of working out such questions as would require two or more applications of the Single Rule of Three.

All questions to which this rule is applicable will be found to contain the statement of some complete case, and an incomplete one in which a quantity corresponding to one of those in the complete case is wanting.

The quantities involved may also be separated into two classes, *causes* and *effects*; and there will be two cases according as it is an effect or a cause which is wanting.

Rule I.—If an effect be wanting.

As product of causes in the complete case : product of causes in the incomplete case : : product of effects in the complete case : product of effects in the incomplete case.

Rule II.—If a cause be wanting.

As product of effects in the complete case : product of effects in the incomplete case : : product of causes in the complete case : product of causes in the incomplete case.

We shall now generally have four terms, but the fourth term will be incomplete, one of its factors being wanting. This will be found by dividing the product of the second and third terms by that of the first and the given part of the fourth (Art. **109**).

Ex. 1. If 28 horses eat 56 bushels of corn in 8 days, how many horses will eat 60 bushels in 12 days?

The complete case here is, that 28 horses will eat 56 bushels of corn in 28 days; but in the other case the number of horses is wanting; also the number of horses and the time are the *causes* on which the quantity of corn eaten depends, which is therefore an effect.

Hence we see that it is a cause which is wanting; we have therefore, by II.—

$$56 : 60 :: 28 \times 8 : 12$$

$$\frac{\cancel{60} \times \cancel{28} \times \cancel{8}}{\cancel{56} \times \cancel{12}} = 20.$$

∴ 20 is the number of horses required.

Ex. 2. If 9 persons spend £120 in 8 months, how much will 24 persons spend in 15 months at the same rate?

Here the number of persons and the time are the causes, and the money spent the effect: an effect is wanting; therefore, by I.—

$$9 \times 8 : 24 \times 15 :: 120$$

$$\frac{\cancel{24} \times \overset{5}{\cancel{15}} \times 120}{\underset{\cancel{3}}{\cancel{9}} \times \cancel{8}} = 600.$$

Ans. £600.

Ex. 3. If 20 masons build a wall 50 ft. long, 2 ft. thick, and 14 ft. high, in 12 days of 7 hours each, in how many days of 10 hours each will 60 masons build a wall 500 ft. long, 4 ft. thick, and 16 ft. high?

Here the number of masons, the number of days, and the number of hours in each day are the causes, and the dimensions of the wall the effects; it is a cause which is wanting; therefore, by II.—

$$50 \times 2 \times 14 : 500 \times 4 \times 16 :: 20 \times 12 \times 7 : 60 \times 10$$

$$\frac{\cancel{500} \times 4 \times 16 \times \cancel{20} \times \cancel{12} \times \cancel{7}}{\cancel{50} \times \cancel{2} \times \cancel{14} \times \cancel{60} \times \cancel{10}} = 64.$$

Ans. 64 days.

Ex. 4. If the carriage of 1 cwt. 3 qrs. 21 lbs. for $52\frac{1}{4}$ miles cost 17*s.* 5*d.*, what will be the cost of the carriage of $2\frac{1}{2}$ tons for $46\frac{1}{2}$ miles?

Here the weight carried and the distance are the causes, and the cost the effect.

Expressing the different quantities as fractions we have

$$1 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lbs.} = 1 \text{ cwt. } 3\tfrac{3}{4} \text{ qrs.} = 1\frac{15}{4 \times 4} \text{ cwt.} = \tfrac{31}{16} \text{ cwt.} = \tfrac{31}{320} \text{ ton}$$

$$17s.\ 5d. = 17\tfrac{5}{12}s. = £\frac{209}{12 \times 20}; \text{ therefore, by I.,}$$

$$\tfrac{31}{320} \times 52\tfrac{1}{4} : 2\tfrac{1}{2} \times 46\tfrac{1}{2} :: \frac{209}{12 \times 20}$$

$$\frac{2\frac{1}{2} \times 46\frac{1}{2} \times \dfrac{209}{12 \times 20}}{\frac{31}{320} \times 52\frac{1}{4}} = \frac{5}{\cancel{2}} \times \frac{\overset{\cancel{3}}{\cancel{93}}}{\cancel{2}} \times \frac{\cancel{209}}{\cancel{12} \times \cancel{20}} \times \frac{\overset{4}{\cancel{320}}}{\cancel{31}} \times \frac{\cancel{4}}{\cancel{209}} = 20.$$

Ans. £20.

Ex. 5. If 12 oxen and 35 sheep eat 12 tons 12 cwt. of hay in 8 days, how much will it cost to feed 9 oxen and 14 sheep for 28 days, the price of hay being 4 guineas a ton, and 3 oxen being supposed to eat as much as 7 sheep?

Here the number of oxen and sheep, and the number of days, are the causes, and the quantity of hay eaten the effect; also, since 12 oxen eat as much as 28 sheep and 9 oxen as 21 sheep, we may express the oxen in terms of sheep, making 63 sheep in the first case and 35 in the second; therefore, to find the quantity of hay consumed, we have, by I.—

$$63 \times 8 : 35 \times 28 :: 12\tfrac{3}{5}$$

$$\text{or } \frac{35 \times 28 \times 12\frac{3}{5}}{63 \times 8} = 35 \times 28 \times \tfrac{63}{5} \times \tfrac{1}{63} \times \tfrac{1}{8}$$

$$\therefore \text{ the cost at 4 guineas a ton} = \overset{7}{\cancel{35}} \times \overset{14}{\cancel{28}} \times \frac{\cancel{63}}{\cancel{5}} \times \frac{1}{\cancel{63}} \times \frac{1}{\underset{\cancel{2}}{\cancel{8}}} \times 4 = 98.$$

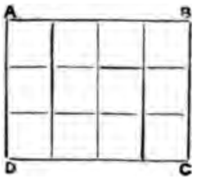
Ans. 98 guineas.

AREAS—SOLID CONTENTS.

114. *Def.*—*Area* is the name given to the *extent* of the *surface* of any figure, and is measured by the number of times it contains a certain fixed area, as a square inch, a square foot, &c.

To find the area of a given rectangle.

Let A B C D be the given rectangle, and suppose A B divided into four equal parts, each say 1 inch in length, and that A D can be exactly divided into three such parts. Through the divisions of A B draw lines parallel to A D or B C, and



through the divisions of A D draw lines parallel to A B or D C. Then the line A B, taken with the first division of the line A D, gives a row of four areas, each 1 sq. inch (Art. **74**); and since A D is divided into three parts, the whole figure A B C D is made up of three such rows, that is, the area of the rectangle A B C D $=4\times3$ or 12 areas, each 1 sq. inch. Hence the following :—

Rule.—Reduce the length and breadth to the same denomination, and then multiply the resulting *numbers* together; the product will be the number of *sq. yards*, *sq. feet*, or *sq. inches* required according as the *linear* dimensions were reduced to yards, feet, or inches.

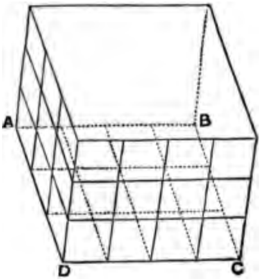
Obs. 1.—The student must notice carefully that by this rule he *does not* multiply inches by inches, feet by feet, &c., but the abstract numbers expressing how many inches, &c., there are in each of the dimensions.

Obs. 2.—Since the area = the length × the breadth, if the area and either dimension be known, the other may be found by dividing the area by the known dimension.

115. *Def.*—*Solid* or *Cubic Content* is the name given to the *bulk* of matter contained in any figure, and is measured by the number of times it contains a certain fixed bulk, as a cubic inch, cubic foot, &c.

To find the solid content of a rectangular solid.

If on each of the square inches in the rectangle A B C D (Art. **114**) we place a cubic inch (Art. **75**), we shall obtain a



solid 1 inch high; and since there are 12 sq. inches in the base, the solid will contain 12 cubic inches. If we repeat the process we shall obtain a solid two inches high, and containing $12 \times 2 = 24$ cubic inches; similarly, if we make three such rows, the solid content will be $12 \times 3 = 36$ cubic inches, and so on. We see, therefore, that to find the solid content, the area of the base must be multiplied by the height; but the area of the base is found by multiplying the length by the breadth. Hence the

RULE.—Reduce the length, breadth, and height to the same denomination, and then multiply the resulting *numbers* together; the continued product will be the number of *cubic yards, cubic feet*, or *cubic inches* required, according as the *linear* dimensions were reduced to yards, feet, or inches.

Obs. 1.—Obs. 1 of Art. **114** applies equally to this rule.

Obs. 2.—Since the solid content = the length × the breadth × the height, if we know the solid content and two of the dimensions, the other may be found by dividing the solid content by the product of the known dimensions.

Ex. 1. Find the area of a room 12 yds. long and 8 yds. 2 ft. wide.

$$8 \text{ yds. } 2 \text{ ft.} = 8\tfrac{2}{3} \text{ yds.} = \tfrac{26}{3} \text{ yds.}$$

$$\therefore \text{ area of room} = 12 \times \tfrac{26}{3} = 104 \text{ s. yds.}$$

Ex. 2. What will be the cost of carpeting a room $9\frac{3}{4}$ yds. long and 4 yards 2 ft. wide with carpet 1 yd. 3 in. wide, at 4*s.* 6*d.* per yard?

The area of the room divided by the width of the carpet will give the number of yards required.

$$= (9\tfrac{3}{4} \times 4\tfrac{2}{3}) \div 1\tfrac{1}{12}$$

$$\therefore \text{ the cost} = \frac{\overset{3}{\cancel{39}}}{\cancel{4}} \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} \times \frac{\cancel{12}}{\cancel{13}} \times \frac{9}{\cancel{2}} = 189s. = £9\ 9s.$$

Ex. 3. How many yards of paper $1\frac{3}{8}$ yd. wide will be required for a room 10 yds. 2 ft. long, 6 yds. 1 ft. wide, and 9 ft. 2 in. high?

To find the area of the walls, we must multiply the *whole length* of wall by the height. Now the whole length of wall will evidently be found by adding together the length and breadth and multiplying by 2. In this case we have therefore

$$(10 \text{ yds. } 2 \text{ ft.} + 6 \text{ yds. } 1 \text{ ft.}) \times 2$$

the area of the wall $\therefore = 17 \times 2 \times \frac{55}{6 \times 3}$; and the number of yards of paper required $\therefore = 17 \times 2 \times \frac{\overset{5}{\cancel{55}}}{\underset{3}{\cancel{6}} \times 3} \times \frac{8}{\cancel{11}} = \frac{680}{9} = 75\frac{5}{9}$ yds. $=$ 75 yds. 1 ft. 8 in.

Ex. 4. What will be the cost of papering a room 14 yds. 1 ft. long, $8\frac{1}{2}$ yds. wide, and 9 ft. 9 in. high, with paper $\frac{3}{4}$ yd. wide at 9*d.* a yard, allowing for 4 windows each 7 ft. by $4\frac{1}{2}$ ft. and 2 doors each $6\frac{3}{4}$ ft. by 5 ft.?

$$(14\tfrac{1}{3} + 8\tfrac{1}{2}) \text{ yds.} \times 2 = 22\tfrac{5}{6} \text{ yds.} \times 2.$$

$$\therefore \text{ area of walls} = \tfrac{137}{6} \times \cancel{2} \times \frac{\overset{13}{\cancel{39}}}{\underset{2}{\cancel{4}} \times \cancel{3}} = \tfrac{1781}{12} = 148\tfrac{5}{12} \text{ s. yds.}$$

$$\text{Area of windows} = \left(\frac{7}{\cancel{3}} \times \frac{\cancel{9}}{\cancel{2} \times \cancel{3}} \times \overset{2}{\cancel{4}}\right) \text{s. yds.} = 14 \text{ s. yds.}$$

$$\text{Area of doors} = \left(\frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{4}} \times \cancel{3}} \times \frac{5}{\cancel{3}} \times \cancel{2}\right) \text{s. yds.} = 7\tfrac{1}{2} \text{ s. yds.}$$

$\therefore$ area of windows + area of doors $= 14$ s. yds. $+ 7\frac{1}{2}$ yds. $= 21\frac{1}{2}$ s. yds.
and area of wall to be papered $= 148\frac{5}{12}$ s. yds. $- 21\frac{1}{2}$ s. yds. $= 126\frac{11}{12}$ s. yds.

$$\text{Length of paper required} = \tfrac{1523}{12} \div \tfrac{3}{4} = (\tfrac{1523}{12} \times \tfrac{4}{3}) \text{ yds.}$$

$$\text{The cost } \therefore = \left(\frac{1523}{\underset{3}{\cancel{12}}} \times \frac{\cancel{4}}{\cancel{3}} \times \cancel{9}\right) d. = 1523d. = £6\ 6s.\ 11d.$$

Ex. 5. Find the solid content of a rectangular block of stone 3 ft. 4 in. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep.

$$3\tfrac{1}{3} \times 2 \times 1\tfrac{1}{2} = \frac{10}{\cancel{3}} \times \cancel{2} \times \frac{\cancel{3}}{\cancel{2}} = 10 \text{ c. feet.}$$

Ans. 10 c. ft.

Ex. 6. Find the solid content of a square cistern whose side measures 3 yds. and depth is 2 ft. 8 in.

$$2 \text{ ft. } 8 \text{ in.} = 2\tfrac{2}{3} \text{ ft.} = \tfrac{8}{3} \text{ ft.} = \frac{8}{3 \times 3} \text{ yds.}$$

$$\text{The content of the cistern} \therefore = 3 \times 3 \times \frac{8}{3 \times 3} = 8 \text{ c. yards.}$$

Ex. 5. A room which contains 5940 c. ft. is 11 yds. long and $7\frac{1}{2}$ yds. wide; find its height.

Reducing the yards to feet and dividing we have for the height

$$\frac{5940}{11 \times 3 \times 7\frac{1}{2} \times 3} = \frac{5940 \times 2}{11 \times 3 \times 15 \times 3} = 8 \text{ feet.} \qquad \textit{Ans.}\ 8 \text{ ft.}$$

DUODECIMALS OR CROSS MULTIPLICATION.

116. This method of working questions in square and cubic measure is employed by painters, bricklayers, &c., in measuring their work.

They measure by feet, inches, parts, &c., decreasing from the left to the right in a twelvefold proportion. Thus 1 foot = 12 inches, 1 inch = 12 parts, and so on: the inches, parts, &c., are called primes, seconds, thirds, &c., and are denoted by one, two, three, &c., accents placed to the right a little above the numbers.

Rule.—Write the terms of the multiplier under the corresponding terms of the multiplicand. Multiply every term in the multiplicand beginning at the lowest by each term of the multiplier successively, beginning at the highest; divide each product which is not of the denomination of feet by 12, add the quotient to the next product, and place the remainder under the term of the multiplicand just used, when the denomination of the multiplier is feet, one place removed to the right when it is primes, two places when it is seconds, and so on. Add the products together, carrying 1 for every 12, and the sum will be the answer.

$$\begin{array}{rrrr}
\text{ft.} & & & \\
5 & 3' & & \\
4 & 9' & & \\
\hline
21 & 0 & & \\
3 & 11 & 3'' & \\
\hline
24 & 11 & 3 & \\
\hline
\end{array}$$

Ans. 24 ft. 11′ 3′.

$24\text{ ft. } 11'\ 3'' = 24\text{ sq. ft.} + (\frac{11}{12} + \frac{3}{144})\text{ sq. ft.} = 24\frac{135}{144}\text{ sq. ft.} = 24\text{ sq. ft. } 135\text{ sq. in.}$

Also $5\frac{1}{4} \times 4\frac{3}{4} = \frac{21}{4} \times \frac{19}{4} = \frac{399}{16}$ sq. ft. $= 24$ sq. ft. 135 sq. in.

Which agrees with the result obtained by cross multiplication.

Ex. 2. Multiply 7 ft. 5 in. 10 pts. by 6 ft. 7 in. 4 pts., and prove the result by vulgar fractions.

$$\begin{array}{rrrrr}
\text{ft.} & & & & \\
7 & 5' & 10'' & & \\
6 & 7 & 4 & & \\
\hline
44 & 11 & 0 & & \\
4 & 4 & 4 & 10''' & \\
 & 2 & 5 & 11 & 4'''' \\
\hline
49 & 5 & 10 & 9 & 4 \\
\hline
\end{array}$$

Ans. 49 ft. 5′ 10″ 9‴ 4⁗

$$49\text{ ft. } 5'\ 10''\ 9'''\ 4'''' =$$

$$49\text{ sq. ft.} + \left(\frac{5}{12} + \frac{10}{12 \times 12} + \frac{9}{12 \times 12 \times 12} + \frac{4}{12 \times 12 \times 12 \times 12}\right)\text{ sq. ft.}$$

$$= 49\text{ sq. ft.} + \frac{8640 + 1440 + 108 + 4}{12 \times 12 \times 12 \times 12}\text{ sq. ft.} = 49\text{ sq. ft.} + \frac{10192}{144 \times 12 \times 12}\text{ sq. ft.}$$

$$= 49\text{ sq. ft.} + \frac{637}{144 \times 9}\text{ sq. ft.}$$

$$= 49\text{ sq. ft.} + \left(\frac{637}{144 \times 9} \times 144\right)\text{ sq. in.} = 49\text{ sq. ft. } 70\tfrac{7}{9}\text{ sq. in.}$$

$$\text{Also } \overset{\text{ft.}}{7}\ \ 5'\ 10'' = 7 + \left(\frac{5}{12} + \frac{10}{12 \times 12}\right)\text{ ft.} = 7\tfrac{35}{72}\text{ ft.} = \tfrac{539}{72}\text{ ft.}$$

$$\overset{\text{ft.}}{6}\ \ 7'\ \ 4'' = 6 + \left(\frac{7}{12} + \frac{4}{12 \times 12}\right)\text{ ft.} = 6\tfrac{11}{18}\text{ ft.} = \tfrac{119}{18}\text{ ft.}$$

$\frac{539}{72} \times \frac{119}{18} = \frac{64141}{1296}$ sq. ft. $= 49$ sq. ft. $70\frac{7}{9}$ sq. in., which agrees with the result obtained by cross multiplication.

Ex. 3. Find the solid content of a cube whose edge is 3 ft. 4 in., and prove the truth of the result by vulgar fractions.

$$\begin{array}{rrrr}
\text{ft.} & & & \\
3 & 4' & & \\
3 & 4' & & \\ \hline
10 & 0 & & \\
1 & 1 & 4' & \\ \hline
11 & 1 & 4 & \\
3 & 4 & & \\ \hline
33 & 4 & 0 & \\
3 & 8 & 5 & 4''' \\ \hline
37 & 0 & 5 & 4
\end{array}$$

Ans. 37 c. ft. 0′ 5″ 4‴

$$37 \text{ c. ft. } 0' \; 5'' \; 4''' =$$

$$37 \text{ c. ft.} + \left(\frac{0}{12} + \frac{5}{12 \times 12} + \frac{4}{12 \times 12 \times 12}\right) \text{ c. ft.} = 37 \text{ ft.} + \frac{60+4}{12 \times 12 \times 12} \text{ c. ft.}$$

$$= 37 \text{ c. ft.} + \tfrac{64}{1728} \text{ c. ft.} = 37 \text{ c. ft. } 64 \text{ c. in.}$$

Also $3\tfrac{1}{3} \times 3\tfrac{1}{3} \times 3\tfrac{1}{3} = \tfrac{10}{3} \times \tfrac{10}{3} \times \tfrac{10}{3} = \tfrac{1000}{27}$ c. ft. = 37 c. ft. 64 c. in. Which agrees with the result obtained by cross multiplication.

INTEREST.

117. *Def.—Interest* is the name given to the money paid for the use of a sum of money for a certain period.

The *Principal* is the money lent.

The *Rate per cent.* is the interest of £100 for one year.

The *Amount* is the principal + the interest.

118. To find the interest of a given sum at a given rate per cent. for a given time.

Rule.—Multiply the principal at the rate per cent. and the time, and divide by 100.

Obs. 1.—It will be noticed that this rule is only an application of the Rule of Three.

Obs. 2.—If the interest be required for any part of a year which cannot be expressed as a convenient fraction of a year, it had better be found by the Rule of Three, from the interest for one year.

Ex. 1. Find the interest of £556 13*s.* 4*d.* for 6 years, at 5 per cent.

£	*s.*	*d.*
556	13	4
		5
2783	6	8
		6
167.00	0	0

Ans. £167.

Ex. 2. Find the interest of £1493 16*s.* 8*d.* for 3 years and 9 months, at 6 per cent.

£	*s.*	*d.*
1493	16	8
		6
8963	0	0
		$3\frac{3}{4}$
26889	0	0
6722	5	0
336.11	5	0
20		
2.25		
12		
3.00		

Ans. £336 2*s.* 3*d.*

Ex. 3. Find the amount of £7689 19*s.* for 723 days, at 4 per cent.

723 days = 1 yr. 358 days.

£	*s.*
7689	19
	4
307.59	16
20	
11.96	
12	
11.52	

£307 11*s.* 11.52*d.* interest, for 1 year.

To find the interest for 358 days we have—

$$\begin{array}{l}
\qquad\qquad\qquad\quad £ \quad s. \quad d.\\
365 : 358 :: 307 \quad 11 \quad 11.52\\
\qquad\qquad\qquad\quad \underline{20}\\
\qquad\qquad\quad\;\; 6151\\
\qquad\qquad\qquad\quad \underline{12}\\
\qquad\qquad\quad 73823.52\\
\qquad\qquad\qquad\quad \underline{3.58}\\
\qquad\qquad 590588.16\\
\qquad\quad 3691176.0\\
\qquad 22147056\\
365)\overline{26428820.16}(72407.72\\
\quad\;\; \underline{2555}\\
\qquad\;\; 878\\
\qquad\;\; \underline{730}\\
\qquad\;\; 1488\\
\qquad\;\; \underline{1460}\\
\qquad\qquad 2820\\
\qquad\qquad \underline{2555}\\
\qquad\qquad\;\; 2651\\
\qquad\qquad\;\; \underline{2555}\\
\qquad\qquad\qquad 966\\
\qquad\qquad\qquad \underline{730}\\
\qquad\qquad\qquad\;\; \underline{36} \qquad\qquad\qquad\qquad 72407.72, \&c
\end{array}$$

Since $365 = 5 \times 73$, if we continued the division we should obtai circulating decimal (Art. 62).

$$\begin{array}{l}
\qquad\qquad d.\\
12)\underline{72407.72}\\
2\backslash0)\underline{603\backslash3} \;.\; 11.72\\
\qquad \underline{301} \;.\; 13
\end{array}$$

$$\begin{array}{l}
\qquad £ \quad s. \quad d.\\
\therefore 301 \quad 13 \quad 11.72 \text{ is the interest for 358 days}\\
\quad\;\; \underline{307 \quad 11 \quad 11.52}\\
\therefore \underline{£609 \quad 5 \quad 11.24} \text{ is the interest for 723 days}
\end{array}$$

$$\begin{array}{l}
\qquad\qquad £ \qquad s. \quad d.\\
\qquad\qquad 7689 \quad 19\\
\qquad\qquad\; \underline{609 \quad\; 5 \quad 11.24}\\
\text{And } \therefore \underline{£8299 \quad 4 \quad 11.24} \text{ is the amount required.}
\end{array}$$

If we had worked for the interest by fractions we should have had

$$£7689\ 19s. = £\tfrac{153799}{20}$$

$$\text{and } \tfrac{153799}{20} \times \tfrac{723}{365} \times 4 \times \tfrac{1}{100} = \tfrac{111196677}{182500} = £609\ 5s.\ 11.235d.$$

119. If the interest be not paid when it becomes due, it is evident that the borrower is using more of the lender's money than he ought for that interest. The interest owing should therefore be added to the principal so as to create a new principal, to be paid for during the ensuing term. When this is done it is called

COMPOUND INTEREST.

120. To find the compound interest of a given sum at a given rate per cent. for a given time.

Rule.—Find the amount for the first year (Art. **117**); consider this as the principal for the second year; the amount for the second year as the principal for the third year, and so on. The compound interest required will be the sum of these successive interests.

The amount will most readily be found by adding the last interest to the last principal.

Ex. 1. Find the compound interest of £270 for 2 years, at 3 per cent.

£	£	s.		£	s.	d.
270	270	0		8	2	0
3	8	2		8	6	10.32
8.10	278	2	*Ans.*	£16	8	10.32
20		3				
2.00	8.34	6				
	20					
	6.86					
	12					
	10.32					

Ex. 2. Find the amount of £180 in 3 years, at 4½ per cent. compound interest.

£
180
4½
720
90
8.10
20
2.00

£	s.
180	0
8	2
188	2
	4½
752	8
94	1
8.46	9
20	
9.29	
12	
3.48	

£	s.	d.
188	2	
8	9	3.48
196	11	3.48
		4½
786	5	1.92
98	5	7.74
8.84	10	9.66
20		
16.90		
12		
10.8966		

	£	s.	d.
	196	11	3.48
	8	16	10.8966
Ans.	£205	8	3.3766

121. In all cases of interest there are four things to be considered:—the principal, the interest, the rate per cent., and the time; and when any three of these are given, the fourth may be found by one of the following rules, which are, however, only particular applications of the Rule of Three.

I. When the principal, interest (or amount), and rate per cent. are given to find the time:—

Rule.—As interest on the principal for 1 year : the given interest : : 1 year.

Obs.—If the amount be given we must subtract the principal from the amount in order to find the interest.

II. When the principal, interest (or amount), and time, are given to find the rate per cent.:—

Rule.—Find the interest on the principal for the given time at any rate (say 1) per cent., then state—

As this interest : the given interest : : 1 per cent.

III. When the rate per cent., interest (or amount), and time, are given to find the principal:—

Rule.—Find the interest on £100 for the given time at the given rate per cent.; then state—

As this interest : the given interest :: £100.

Or if the amount be given :—

As the amount of £100 : the given amount :: £100.

The remaining case in which the principal, time, and rate per cent. are given to find the interest (or amount) has been considered (Art. **118**).

Ex. 1. In what time will £546 13*s.* 4*d.* amount to £612 5*s.* 4*d.* at $3\frac{3}{4}$ per cent.?

The interest on the principal for 1 year $= 546\frac{2}{3} \times 3\frac{3}{4} \times \frac{1}{100}$

$$= \frac{\overset{41}{\cancel{1640}}}{\cancel{3}} + \frac{\overset{5}{\cancel{15}}}{\cancel{4}} \times \frac{1}{\underset{2}{\cancel{100}}} = \frac{41}{2} = £20\ 10s.$$

The given interest = £612 5*s.* 4*d.* − £546 13*s.* 4*d.* = £65 12*s.* 0*d.*

∴ by I., £20 10*s.* : £65 12*s.* :: 1 year

$$20\tfrac{1}{2} : 65\tfrac{3}{5} :: 1;\ \frac{2}{\cancel{41}} \times \frac{\overset{8}{\cancel{328}}}{5} \times 1 = \tfrac{16}{5}\text{ yr.} = 3\tfrac{1}{5}\text{ yr.} = 3\text{ years }75\text{ days.}$$

Ans. 3 years 75 days.

Ex. 2. At what rate per cent. will the interest £539 6*s.* 8*d.* amount to £109 4*s.* $3\frac{3}{5}d.$ in $4\frac{1}{2}$ years?

£	*s.*	*d.*
539	6	8
		$4\frac{1}{2}$
2157	6	8
269	13	4
24.27	0	0

Hence the interest on the principal for the given time, at 1 per cent., is £24.27.

∴ by II., £24.27 : £109 4*s.* $3\frac{3}{5}d.$: 1.

and 4*s.* $3\frac{3}{5}d.$ = £.215 ∴ £24.27 : £109.215 :: 1.

$$\begin{array}{r} 24.27)109.215(45 \\ \underline{97.08} \\ 12.135 \\ \underline{12.135} \end{array} \qquad 4.5 = 4\tfrac{1}{2}$$

∴ $4\frac{1}{2}$ is the rate per cent. required.

Ex. 3. What sum of money will amount to £193 2*s.* 6*d.* in 9 months at 4 per cent. per annum ?

In the given time at the given rate £100 would amount to £103, ∴ by III., £103 : £193 2*s.* 6*d.* :: £100.

$$\pounds 103 : \pounds 193\tfrac{1}{8} :: \pounds 100\ \frac{1}{\cancel{103}} \times \frac{\overset{15}{\cancel{1545}}}{\underset{2}{\cancel{8}}} \times \overset{25}{\cancel{100}} = \frac{375}{2} = \pounds 187\ 10s.$$

£187 10*s.* is therefore the principal required.

DISCOUNT.

122. We have seen that *interest* is the money paid by the borrower for the use of a sum of money for a certain period. On the contrary—

Def.—Discount is the money allowed for the payment of a sum of money before it becomes legally due.

For instance, if A has to pay B £105 a year hence, and B requires the money at once, it is evident that if A were to give B £105 he would be paying him too much. For if money be worth, say, 5 per cent., B, by lending the £105 to some one else, would receive £5 5*s.* interest at the end of the year, and would therefore then have £110 5*s.*, that is, £5 5*s.* more than was due to him, while B would be a loser to that amount. It is plain, therefore, that A ought to pay B only £100, because £100, with its interest, would at the end of the year amount to £105, the sum then due to B. Here £5, the sum allowed to A, is the *discount*; and £100, the money paid by A, is called the *present worth* or *present value* of £105 due a year hence at £5 per cent.: hence we have the following definition :—

Def.—Present worth is that sum of money which would amount to the sum due in the given time at the given rate per cent.

All questions, therefore, relating to present worth fall under III. (Art. **121**); and the discount being the difference between the present worth and the sum due, may be found by subtracting the former from the latter; but if only the discount be required, it will be better to use the following rule, which gives it at once.

RULE.—Find the amount of £100 for the given time at the given rate per cent., then state—

As this amount : the given sum : : the interest of £100.

Obs.—It will be noticed, therefore, that the only difference in stating for present worth and discount is, that for the former we put £100 in the third term, and for the latter the *interest* on £100.

123. The most common form in which discount occurs is in the prepayment of Bills of Exchange. These are orders to pay a certain sum of money at a future date, and if the credit of the parties whose names are on the bill be good, a banker will discount it, that is, he will pay to the holder its present value, receiving himself the full amount when the bill becomes due. In practice, however, instead of the *true discount*, the banker deducts the *interest* on the full amount for the time which must elapse before the bill becomes due. The holder of the bill will therefore receive a little less than its true present worth, the banker gaining the difference by the transaction. Thus, in the example given in Art. **122**, the banker would deduct £5 5*s.* instead of the true discount £5, the holder of the bill receiving £99 15*s.* instead of £100, and the banker gaining 5*s.* by the transaction.

124. In Great Britain three days, called days of grace, are allowed after a bill becomes nominally due before it is legally due, and these must therefore be allowed for in calculating the discount.

If a bill would fall nominally due on the 31st of any month which has only thirty days, it is considered as falling due on the last day of that month, and will therefore be legally due on the 3rd of the following month. Under similar circumstances bills are considered due on the *last* day of February.

If a bill fall legally due on Sunday it must be paid in England on the Saturday, but in Ireland on the Monday.

Ex. 1. Find the discount on £189 15*s*. for 8 months at $3\frac{1}{3}$ per cent.

$$\tfrac{2}{3} \text{ of } 3\tfrac{1}{3} = \tfrac{2}{3} \text{ of } \tfrac{10}{3} = \pounds\tfrac{20}{9} = \text{interest of } \pounds 100 \text{ for 8 months}$$

$$\therefore \text{ by Rule, } 100\tfrac{20}{9} : 189\tfrac{3}{4} :: \tfrac{20}{9}$$

$$\tfrac{9}{920} \times \tfrac{759}{4} \times \tfrac{20}{9} = \tfrac{759}{184} = 4\tfrac{23}{184} = \pounds 4\tfrac{1}{8} = \pounds 4\ 2s.\ 6d.$$

$\therefore$ £4 2*s*. 6*d*. is the discount required.

Ex. 2. Find the discount on £487 for 5 months at $3\frac{1}{2}$ per cent.

$$\tfrac{5}{12} \text{ of } 3\tfrac{1}{2} = \tfrac{5}{12} \text{ of } \tfrac{7}{2} = \pounds\tfrac{35}{24} = \text{interest of } \pounds 100 \text{ for 5 months}$$

$$\therefore 100\tfrac{35}{24} : 487 :: \tfrac{35}{24}$$

$$\tfrac{24}{2435} \times 487 \times \tfrac{35}{24} = \pounds 7$$

$\therefore$ £7 is the discount required.

Ex. 3. Find the present worth of £676 13*s*. 4*d*., due 6 months hence, at 3 per cent.

$$\tfrac{1}{2} \text{ of } 3 = \pounds\tfrac{3}{2} = \text{interest of } \pounds 100 \text{ for 6 months}$$

$$\therefore \text{ by Rule } (\mathbf{121}) \text{ III., } 100\tfrac{3}{2} : 676\tfrac{2}{3} :: 100$$

$$\tfrac{2}{203} \times \tfrac{2030}{3} \times 100 = \pounds\tfrac{2000}{3} = \pounds 666\ 13s.\ 4d.$$

$\therefore$ £666 13*s*. 4*d*. is the present worth required.

Ex. 4. A bill for £219, drawn February 15, at 10 months, is discounted by a banker on March 3, at 5 per cent.; how much will the banker gain?

The bill will fall legally due December 18, and from March 3 to December 18, the last inclusive, is 290 days.

The interest of £219 for 290 days $= 219 \times \frac{290}{365} \times 5 \times \frac{1}{100} = \pounds\frac{87}{10} = \pounds 8$ 14*s*.

For the true discount we have: Interest of £100 for 290 days $= \frac{290}{365}$ of $5 = \frac{290}{73}$: $\therefore 100\frac{290}{73} : 219 :: \frac{290}{73}$.

$$\tfrac{73}{7590} \times 219 \times \tfrac{290}{73} = \tfrac{2117}{253} = \pounds 8\ 7s.\ 4\tfrac{56}{253}d.$$

$$\begin{array}{rrr} \pounds & s. & d. \\ 8 & 14 & 0 \\ 8 & 7 & 4\frac{56}{253} \\ \hline & 6 & 7\frac{197}{253} \\ \hline \end{array}$$

The banker will therefore gain 6*s*. $7\frac{197}{253}$*d*.

COMMISSION, BROKERAGE, INSURANCE.

125. *Def.*—*Commission* is the name given to the sum of money allowed to an agent for buying or selling goods for another person.

Brokerage is the commission allowed to a person called a broker, whose business it is to buy or sell stock, shares, &c., for others.

Insurance is a contract entered into for the payment of a fixed sum of money if a certain specified event happen. The money paid to insure this is called the *premium*, and the legal document by which payment is secured is called the *policy of insurance.*

In all these cases, since the commission, &c., is usually so much per cent., it may be found by the same rule as simple interest.

There is, however, one particular case of insurance which may be noticed.

It is sometimes required to find for what amount goods worth a certain sum must be insured, so that in the event of loss the party insuring may recover the value of his goods, together with the amount of premium paid. To do this subtract the rate per cent. from £100, and then state—

As this remainder : value of goods : : £100.

It may here be noticed that tradesmen frequently allow what is commonly called a *discount* for ready money. Thus, if 5 per cent. discount be allowed for cash, since 5 per cent. is 1*s.* in the pound, a person who purchased goods to the amount of £8 10*s.*, would have to pay only £8 1*s.* 6*d.* The discount being 8*s.* 6*d.*, that is, 8*s.* for the £8 and 6*d.* for the 10*s.*

It will be noticed that no account is taken of the time.

Ex. 1. How much will a commission agent receive for selling £745 worth of goods at 7½ per cent.?

$$\begin{array}{r}
\pounds \\
745 \\
7\frac{1}{2} \\
\hline
5215 \\
372\frac{1}{2} \\
\hline
55.87\ 10s. \\
20 \\
\hline
17.50 \\
12 \\
\hline
6.00 \\
\hline
\end{array}$$

Ans. £55 17*s.* 6*d.*

Ex. 2. For what sum should goods worth £68 19*s.* be insured at 1½ per cent., so that in the event of loss the owner may recover the value of the goods together with the premium paid?

At this rate £100 would cover goods worth £98 10*s.*, together with the premium; we have therefore

$$\begin{array}{c}
\pounds\ \ s.\ \ \ \pounds\ \ s. \\
98\ 10 : 68\ 19 :: 100 \\
98\frac{1}{2} : 68\frac{19}{20} :: 100 \\
\frac{2}{197} \times \frac{1379}{20} \times 100 = \pounds 70.
\end{array}$$

Ans. £70.

STOCKS.

126. *Stock* is the name given to the capital of a trading company, and also to the money lent to government, and for which government engages to pay a certain rate of interest.

A person holding stock has his name entered in the books of the company or of the government, as the case may be, for the amount of stock he holds.

The profits are usually divided amongst the shareholders every half year, and the amount which each receives is called his dividend.

If a holder of stock wants his money, the company or government will not repay him the money he has lent, but if he can find a purchaser he may sell his stock like any stituted for his in the books.

If the business of the company be prosperous, the profits, after paying all necessary expenses, may be considerable, and then the dividends will be large, perhaps 10, 15, or 20 per cent.

In such a case, if a shareholder wishes to sell out, and money be worth, say 5 per cent. in the money market, it is evident he will ask more than £100 for every £100 stock, because for that he receives, say £15 a year, whereas £100 would only bring him in £5. The money he would be able to get for every £100 stock (the actual amount of which will be influenced by many considerations), is called the *price* of the company's stock at that particular time. If, however, the business of the company be not so prosperous, and the dividend less than the rate per cent. which can be obtained for money on other securities, the shareholder will receive less than £100 for every £100 stock.

When £100 stock will sell for more than £100, the stock is said to be at so much *premium.*

When £100 stock will sell for exactly £100, the stock is said to be at *par.*

When £100 stock sells for less than £100, the stock is said to be at so much *discount.*

127. Since the interest paid by government is generally less than that which can be obtained on other securities, goverment stock is usually at a discount. For if 5 per cent. can be obtained for money while government pays only 3 per cent., it is evident that no one will give £100 for the privilege of receiving £3 a year, when he can get £5 a year for the same money. Thus, when government stock is quoted in the money market at $91\frac{5}{8}$, a person having £100 stock, on selling out, would receive only £91 12*s.* 6*d.*, and for this sum the purchaser would have his name entered in the government books as the holder of £100 stock, and would receive interest for that amount.

Government stock is commonly known as the *funds*, from the fact of the interest being paid out of the public property or funds, which consists of the receipts from taxes, &c.

128. All questions relating to stock are examples of the Rule of Three; but the best method of treating them will be shown in the following examples :—

Ex. 1. How much 3 per cent. stock can be bought by investing £1023 at 93?

Every £93 *cash* will buy £100 *stock* ∴ the amount of stock = $\frac{1023}{93} \times 100 =$ £1100 stock.

Ex. 2. Find the value in money of £2240 $3\frac{1}{4}$ per cent. stock at $81\frac{7}{8}$. Here every £100 *stock* will give £$81\frac{7}{8}$ *cash*, ∴ the value $= \frac{2240}{100} \times 81\frac{7}{8}$ $= \frac{2240}{100} \times \frac{655}{8} =$ £1834.

Ex. 3. What will be my income if I invest £1638 in the $4\frac{1}{2}$ per cents. at $93\frac{3}{5}$?

For every £$93\frac{3}{5}$ *cash* I can buy £100 *stock*, for which I shall receive £$4\frac{1}{2}$ a year. Hence my income

$$= \frac{1638}{93\frac{3}{5}} \times 4\frac{1}{2} = \frac{1638 \times 5}{468} \times \frac{9}{2} = £\tfrac{315}{4} = £78\ 15s.$$

Ex. 4. What sum of money must be invested in the 3 per cents. at $91\frac{3}{4}$ to produce an income of 100 guineas?

For every £3 income £100 stock must be bought, and this will cost £$91\frac{3}{4}$; therefore the money that must be invested

$$= \tfrac{105}{3} \times 91\tfrac{3}{4} = \tfrac{105}{3} \times \tfrac{367}{4} = \frac{35 \times 367}{4} = \tfrac{12845}{4} = £3211\ 5s.$$

Ex. 5. At what rate per cent. will a person receive interest who invests in the $3\frac{1}{2}$ per cents. at 91?

This is the same as the question—If a person gets £$3\frac{1}{2}$ for £91, what will he get for £100 ?

$$91 : 100 : 3\tfrac{1}{2} \quad \frac{100 \times 7}{91 \times 2} = 3\tfrac{11}{15}.$$

Ans. $3\frac{11}{15}$ per cent.

brokerage being charged $\frac{1}{8}$ per cent.?

With the brokerage, the price of the stock to the purchaser will be $90\frac{3}{8}+\frac{1}{8}=90\frac{1}{2}$ ∴ the amount of stock $=\dfrac{2353}{90\frac{1}{2}}\times 100=$ £2600 stock.

Ex. 7. A person transfers £1000 stock from the 4 per cents. at 90, to the 3 per cents. at 72. Find the alteration in his income.

His income in the 4 per cents. $=\frac{1000}{100}\times 4=$ £40.

Selling out at 90 he gets $\frac{1000}{100}\times 90$ cash.

Investing this at 72 he gets $\dfrac{\frac{1000}{100}\times 90}{72}\times 100$ stock $=\dfrac{1000\times 90}{100\times 72}\times 100$

and the income derived from this, at 3 per cent.

$=\dfrac{1000\times 90}{100\times 72}\times 100\times\frac{1}{100}\times 3=\frac{75}{2}=$ £37 10*s.*; and £40 − £37 10 = £2 10*s.*

His income will therefore be less by £2 10*s.*

Ex. 8. A person buys £500 stock at $98\frac{3}{4}$ and sells out at 103. What does he gain by the transaction?

The money invested $=\frac{500}{100}\times 98\frac{3}{4}=$ £493 15*s.*

The money obtained by selling out $=\frac{500}{100}\times 103=$ £515.

He therefore gains £515 − £493 15*s.* = £21 5*s.*

PROFIT AND LOSS.

129. Questions in Profit and Loss are worked by the Rule of Three, but a little ingenuity is sometimes required in stating them.

Ex. 1. An article which cost 3*s.* 6*d.* is sold for 3*s.* $10\frac{1}{2}d.$; what is the profit per cent.?

The gain in 3*s.* 6*d.* is $4\frac{1}{2}d.$, ∴ for the gain in £100, we have

$$3s.\ 6d. : £100 :: 4\tfrac{1}{2}d. \qquad \text{or } \frac{7}{2\times 20} : 100 :: \frac{9}{2\times 12\times 20}$$

$$\frac{2\times 20}{7}\times 100\times\frac{9}{2\times 12\times 20}=\frac{75}{7}=10\tfrac{5}{7}\text{ per cent.}$$

Ex. 2. For how much must an article which cost 16*s.* 6*d.* be sold so as to gain $33\frac{1}{3}$ per cent.?

To gain $33\frac{1}{3}$ per cent., an article which cost £100 must be sold for £$133\frac{1}{3}$; we have therefore

$$£100 : 16s.\ 6d. :: £133\tfrac{1}{3}$$

$$\tfrac{1}{100} \times \frac{33}{2 \times 20} \times \tfrac{400}{3} = £\tfrac{22}{20} = £1\ 2s.$$

Ex. 3. An article which cost £1 5*s.* is sold at a loss of 10 per cent.; what price did it fetch?

To lose 10 per cent. an article which cost £100 must be sold for £90; we have therefore

$$£100 : £1\ 5s. :: £90$$

$$\tfrac{1}{100} \times \tfrac{5}{4} \times 90 = \tfrac{9}{8} = £1\ 2s.\ 6d.$$

Ex. 4. What was the prime cost of an article which, when sold for £1 7*s.*, realised a profit of 8 per cent.?

To realise 8 per cent., what cost £100 must be sold for £108; we have therefore

$$£108 : £1\ 7s. :: 100$$

$$\tfrac{1}{108} \times \tfrac{27}{20} \times 100 = £\tfrac{5}{4} = £1\ 5s.$$

If it had been sold at a loss of 8 per cent. we should have had 100 − 8 or 92 in the first term.

Ex. 5. If 5 per cent. be lost by selling an article for 5*s.*, for how much must it be sold so as to gain $4\frac{1}{2}$ per cent.?

An article which cost £100 must be sold for £95 to lose 5 per cent., and for £$104\frac{1}{2}$ to gain $4\frac{1}{2}$ per cent.; we have therefore

$$£95 : 5s. : £104\tfrac{1}{2}$$

$$\tfrac{1}{95} \times \tfrac{5}{20} \times \tfrac{209}{2} = £\tfrac{11}{40} = 5s.\ 6d.$$

Ex. 6. By selling an article for £51 16 per cent. is lost; how much per cent. is gained or lost by selling it for £76?

To find the prime cost we have, Ex. 4—

$$£84 : £51 :: £100, \text{ which gives } £\tfrac{425}{7}$$

$$£76 - £\tfrac{425}{7} = £\tfrac{107}{7} = \text{gain by selling it for } £76.$$

∴ by Ex. 1, £$\frac{425}{7}$: 100 :: $\frac{107}{7}$, from which we find the gain per cent. to be £$25\frac{3}{17}$.

sugar at 9½*d.* per lb. so as to gain 5 per cent. by the exchange?

To gain 5 per cent. on every 4*s.* 9*d.* we have for the price of the tea per lb.: £100 : 4*s.* 9*d.*::105, which gives $\frac{19 \times 105}{4 \times 20 \times 100}$; and 28 lbs. at 9½*d.* $= £\frac{19 \times 28}{12 \times 20 \times 2}$.

∴ dividing by the price of the tea per lb.

$$\frac{19 \times 28}{12 \times 20 \times 2} \times \frac{4 \times 20 \times 100}{19 \times 105} = \frac{40}{9} = 4\frac{4}{9}$$

∴ $4\frac{4}{9}$ lbs. is the quantity of tea which must be given.

PROPORTIONAL PARTS.

130. To divide a given quantity into parts which shall have to each other certain given ratios.

Rule.—Divide the quantity by the sum of the numbers expressing the given ratios, and multiply the quotient by each of these numbers separately; the products so formed will be the parts required.

Obs.—If the given ratios be expressed by fractional quantities, these must first be reduced to a common denominator, and the given quantity then be divided into parts which have the same ratios as the new numerators.

The reason of this rule is evident. The parts being represented by fractions having a common denominator are in the same ratio as the numerators, that is, as the given numbers, and since the denominator is the sum of the numerators, the sum of the fractions is unity, i. e. the sum of the parts make up the whole quantity.

Questions in partnership, and the like, are worked by this rule.

Ex. 1. Divide 1065 into parts which shall be to each other in the ratio of 3, 5, 7, and also into parts which shall be in the ratio of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$.

$3 + 5 + 7 = 15$, and the parts are $\frac{3}{15}$, $\frac{5}{15}$, and $\frac{7}{15}$ of 1065

$\frac{1065}{15} = 71$; $71 \times 3 = 213$; $71 \times 5 = 355$; $71 \times 7 = 497$

∴ the parts required are 213, 355, 497.

Again $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}=\frac{35}{105}$, $\frac{21}{105}$, $\frac{15}{105}$ respectively

$35+21+15=71$; $\frac{1065}{71}=15$; $15\times35=525$; $15\times21=315$; $15\times15=225$

∴ the parts required are 525, 315, 225.

Ex. 3. A company of militia, consisting of 104 men, is to be raised from three towns, the population of which is 1400, 4200, and 3500 respectively. How many must be sent from each town?

Here 104 is to be divided into parts which are in the ratio of 1400, 4200, and 3500, i. e. of 2, 6, 5.

$2+6+5=13$; $\frac{104}{13}=8$; $8\times2=16$; $8\times6=48$; $8\times5=40$

∴ the numbers required are 16, 48, and 40.

Ex. 3. A., B., and C. engage in trade; A. invests £500, B. £750, and C. £350. The profits amount to £1153 12*s.*; what should each receive?

Here £1153 12*s.* is to be divided into parts which shall have the ratio of 500, 750, and 350; i. e. of 10, 15, and 7; $10+15+7=32$

$\frac{£1153\ 12s.}{32}=£36\ 1s.$; $£36\ 1s.\times10=£360\ 10s.$; $£36\ 1s.\times15=£540\ 15s.$; $£36\ 1s.\times7=£252\ 7s.$ Their respective shares are therefore £360 10*s.*, £540 15*s.*, and £252 7*s.*

Ex. 4. A., B., C., and D. engage in trade; A. puts in £300 for 12 months, B. £330 for 10 months, C. £375 for 8 months, and D. £395 for 6 months. The whole gain is £1227; how much should each receive?

A.'s interest is £300 for 12 months, which is the same as £300 × 12, or £3600 for 1 month.

B.'s interest is £330 for 10 months, which is the same as £330 × 10, or £3300 for 1 month.

C.'s interest is £375 for 8 months, which is the same as £375 × 8, or £3000 for 1 month.

D.'s interest is £395 for 6 months, which is the same as £395 × 6, or £2370 for 1 month.

Their respective shares should therefore be in the ratio of 3600, 3300, 3000, and 2370; i. e. of 120, 110, 100, and 79

$120+110+100+79=409$; $£\frac{1227}{409}=£3$; $£3\times120=£360$;

$£3\times110=£330$; $£3\times100=£300$; $£3\times79=£237$. The shares are therefore £350, £330, £300, and £237.

and £389 8s. 10d. respectively?

Here we have to divide £456 6s. 8d. in the ratio of the 3 quantities £97 7s. 2½d., £292 1s. 7½d., and £389 8s. 10d., which if reduced to the same denomination will be found to be as 1, 3, 4.

$1+3+4=8$; $\frac{£456\ 6s.\ 8d.}{8}=£57\ 0s.\ 10d.$; $£57\ 0s.\ 10d. \times 1=£57\ 0s.\ 10d.$; $£57\ 0s.\ 10d. \times 3=£171\ 2s.\ 6d.$; $£57\ 0s.\ 10d. \times 4=£228\ 3s.\ 4d.$ They will therefore receive £57 0s. 10d., £171 2s. 6d., and £228 3s. 4d. respectively.

EXCHANGE.

131. *Def.—Exchange* is the method by which we determine how much money of one country must be given for a fixed sum of the money of another country according to the *course of exchange* existing at the time.

The *Course of Exchange* is the variable sum of the money of one place which is equivalent to a fixed sum of the money of another place. Thus in London £1 sterling is given for a variable number of French francs, more or less, according to circumstances.

The *Par of Exchange* is the intrinsic value of the money which is equal to that fixed sum.

Agio is the difference per cent. in some foreign places between the value of the bank and the current money; the former being purer than the latter.

Usance is the usual time allowed for the payment of Bills of Exchange, and varies in different countries.

Ex. 1. A merchant in Paris draws a bill for 4860 francs on a merchant in London. How much sterling money will the latter have to pay, exchange being £1 = 24·3 francs.

24·3 fr. : 4860 fr. :: £1. *Ans.* £200.

Ex. 2. What is the course of exchange between London and Lisbon when 1512 milrees are received for £315?

1512 milr. : 1 milr. : : £315.

Ans. 1 milree = 50*d.*

132. *Arbitration* or *Comparison* of exchanges is the method by which we determine the rate of exchange called the *par of arbitration*, between the first and last of a given number of places, when the course of exchange between the first and second, the second and third, &c., is known.

When the par of arbitration is known, a person knows, by comparing it with the present course of exchange, how to remit his money with the greatest advantage.

Arbitration is called simple or compound, according as three or more places are concerned.

Ex. 1. What is the par of arbitration between London and Paris when 3 francs = 480 rees, 400 rees = $3\frac{1}{2}$*s.* Flemish, and 35*s.* Flemish = £1?

$$£1 = \left(\frac{35}{3\frac{1}{2}} \times 400\right)\text{rees} = \left(\frac{35}{3\frac{1}{2}} \times 400 \times \frac{1}{480} \times 3\right)\text{francs} = 25 \text{ francs.}$$

Ans. £1 = 25 francs.

EQUATION OF PAYMENTS.

133. *Def.—Equation of Payments* is the method of finding the proper time for the payment in one sum of two or more sums of money due at different times.

It is evident that the interest of the sums due before the equated time should be equal to the discount of the sums due after that time; but in practice it is usual to make the interest of the sums due before the equated time equal to the interest of those due after it.

Rule.—Multiply each debt into the time which must elapse before the debt becomes due. The sum of these products divided by the sum of the debts will give the equated time.

$$\frac{400 \times 3 + 600 \times 8}{1000} = \frac{1200 + 4800}{1000} = 6.$$

Ans. 6 months.

Ex. 2. A person has to pay £200 in 2 months, £100 in 3½ months, and £150 more in 5 months; when ought he to pay the whole in one sum?

$$\frac{200 \times 2 + 100 \times 3\frac{1}{2} + 150 \times 5}{450} = \frac{400 + 350 + 750}{450} = \frac{1500}{450} = 3\frac{1}{3}.$$

Ans. 3⅓ months.

SQUARE ROOT.

134. To extract the square root (Art. **15**) of a given number.

Rule.—Place a point over the units' place, and then over every second figure to the left, thus dividing the number into periods, each of which corresponds to a figure in the root.

Subtract from the first period the greatest square number it contains, and place its root a little to the right of the number in the form of a quotient. This will be the first figure of the root. To the remainder bring down the next period.

Multiply the part of the root already found by 2, and place the product as a trial divisor a little to the left of the remainder.

See how many times this will go omitting the last figure of the remainder, and place the quotient so found in the root and also on the right of the trial divisor. Multiply the number so formed by the last figure in the root, and subtract the product from the remainder. To the new remainder bring down the next period, multiply the part now found of the root by 2 for a new trial divisor, and proceed as before.

The *reason* for this rule cannot be fully understood by the student, unless he has some knowledge of Algebra.

Ex. 1. Extract the square root of 399424.

```
     39̇94̇24̇(632
     36
 123 | 394
     | 369
1262 | 2524
     | 2524          ∴ √39424 = 632.
```

Here 36 is the greatest square number in 39; 6 is ∴ the first figure of the root, and subtracting 36, the remainder together with the next period gives 394. Multiplying the part now found of the root by 2 we get 12 for the trial divisor; this will go into 39 3 times, we ∴ set 3 in the root and also on the right of the trial divisor, making 123; multiplying this by 3 we get 369, which when subtracted leaves 25. To this remainder we bring down the next period, and multiply 63, the part now found of the root, by 2 for the new trial divisor 126. This gives 2 for the next figure of the root; we ∴ set 2 in the root and also to the right of the trial divisor 126, making 1262. Multiplying this by 2, the last figure in the root, and subtracting, there is no remainder; and we therefore know that the root has been exactly found.

136. It will sometimes happen that the figure given by the trial divisor will be found to be too large. Whenever this is the case the next smaller figure should be tried.

If at any step the trial divisor will not go into the remainder omitting the last figure, a cypher must be put in the root, and the next period brought down.

Ex. 1. Extract the square root of 824464.

```
       82̇44̇64̇(908
       81
  1808)14464
       14464          Ans. 908.
```

136. If the proposed number be a decimal, the square root will be found by the same process, but we must be very careful to begin pointing from the *units'* place, then over second figure to the left and right. The number the number of decimal places in the root.

Ex. 3. Extract the square root of 167·9616.

$$\begin{array}{r|l}
 & 16\dot{7}\cdot 9\dot{6}1\dot{6}(1296 \\
 & 1 \\
22 & 67 \\
 & 44 \\
249 & 2396 \\
 & 2241 \\
2586 & 15516 \\
 & 15516
\end{array}$$

Ans. 12·96.

137. If the proposed number be not a perfect square, its root cannot be exactly found, but we may approximate to it as nearly as we please by annexing cyphers for decimal places.

Ex. 4. Extract the square root of ·025 to four places of decimals.

$$\begin{array}{r|l}
 & \cdot 0\dot{2}5\dot{0}(1581 \\
 & 1 \\
25 & 150 \\
 & 125 \\
308 & 2500 \\
 & 2464 \\
3161 & 3600 \\
 & 3161 \\
 & 439
\end{array}$$

Ans. ·1581.

138. To extract the square root of a vulgar fraction, it the numerator and denominator are both square numbers find the root of each separately, and the answer will thus be obtained as a vulgar fraction; but if the numerator and denominator are not square numbers, the fraction must first be reduced to a decimal, when the root can be found as above.

Ex. 5. Extract the square root of $\frac{169}{625}$.

$\sqrt{169}=13$

$\sqrt{625}=25$

$\therefore \sqrt{\frac{169}{625}}=\frac{13}{25}$ *Ans.*

CUBE ROOT.

139. To extract the cube root (Art. 15) of a given number.

Rule.—Place a point over the units' place, and then over every third figure to the left, thus dividing the number into periods, each of which corresponds to a figure in the root. Subtract from the first period the greatest cube number it contains, and place its root a little to the right of the number in the form of a quotient. This will be the first figure of the root.

To the remainder bring down the next period.

Multiply the part of the root already found by 3, and place the product some little distance to the left of the remainder.

Next square the part of the root already found, multiply this by three, annex two cyphers to the product, and place the number so formed as a trial divisor between the last number and the remainder.

See how many times this will go into the remainder, and place the figure so found in the root and also on the right of the number *first* placed to the left of the remainder.

Multiply the number so formed by the last figure in the root, and add the product to the trial divisor.

Now multiply this sum by the last figure in the root, and subtract the product from the remainder.

To the new remainder (if any) bring down the next period, and proceed exactly as before.

The reason for this rule cannot be fully understood by the student unless he has some knowledge of Algebra.

Ex. 1. Extract the cube root of 681472.

$$\begin{array}{rr|l} & & 68\dot{1}47\dot{2}(88 \\ & & 512 \\ \hline 248 & 19200 & 169472 \\ & 1984 & \\ \hline & 21184 & 169472 \\ \hline \end{array}$$

∴ $\sqrt[3]{681472} = 88$.

together with the next period, gives 162472. Multiplying the part of the root now found by 3 we get 24, which we place some little distance to the left of the remainder. Next squaring the part of the root now found gives 64, and 64 × 3 = 192. To this we annex 2 cyphers and place it as a trial divisor between 24, the number first formed, and the remainder. We see that 19200 will go 8 times into 162472; we therefore set 8 in the root and also on the right of the 24. Multiplying the 248 so formed by 8 and adding the product to the trial divisor, gives 21184. Now this multiplied by 8, the last figure of the root, gives 169472, which when subtracted leaves no remainder, and we therefore know that the root has been exactly found.

140. As in finding the square root, it will sometimes happen that the figure given by the trial divisor will be found to be too large, and then the next smaller figure should be tried.

If at any step the trial divisor will not go into the remainder, a cypher must be put in the root, the next period brought down, and two cyphers added to the trial divisor.

If the proposed number be a decimal the cube root will be found by the same process, but we must be very careful to begin pointing from the *units'* place, and then over every third figure to the left and right. The number of points to the right of the units' place will be the same as the number of decimal places in the root.

If the proposed number be not a perfect cube its root cannot be *exactly* found, but we may approximate to it as nearly as we please by adding cyphers for decimal places.

Ex. 2. Extract the cube root of 95443·993.

		95443·993(45·7
		64
125	4800	31443
	625	
	5425	27125
1357	607500	4318993
	9499	
	616999	4318993

Ans. 45·7.

Ex. 3. Extract the cube root of ·000912673.

$$\begin{array}{rr|l}
\multicolumn{3}{r}{\cdot 00\dot{0}91\dot{2}67\dot{3}(97} \\
\multicolumn{3}{r}{729} \\
\cline{3-3}
277 & 24300 & 183673 \\
 & 1939 & \\
\cline{2-2}
 & 26239 & 183673 \\
\cline{2-3}
\end{array}$$

Ans. ·097.

Ex. 4. Extract the cube root of ·2 to three places of decimals.

$$\begin{array}{rr|l}
\multicolumn{3}{r}{\cdot 20\dot{0}(584} \\
\multicolumn{3}{r}{125} \\
\cline{3-3}
158 & 7500 & 75000 \\
 & 1264 & \\
\cline{2-2}
 & 8764 & 70112 \\
\cline{2-3}
1744 & 1009200 & 4888000 \\
 & 6976 & \\
\cline{2-2}
 & 1016176 & 4064704 \\
\cline{2-3}
 & & 823296
\end{array}$$

Ans. ·584.

141. Questions are sometimes proposed which may appear not to fall under any of the preceding rules. Many of these, however, will be found to depend on the principles of proportion; and if the student has mastered the preceding pages, he will experience but little difficulty in dealing with them.

A few miscellaneous examples of this kind will now be worked out by way of illustration.

Ex. 1. A gentleman who played in eight cricket matches during the season made the following scores, 35, 15, 17, 40, 3, 11, 21, and 18; what was his average score?

The total number of runs $= 35 + 15 + 17 + 40 + 3 + 11 + 21 + 18 = 160$, and the number of matches was 8; $\therefore$ the average $= 160 \div 8 = 20$.

Ans. 20.

Ex. 2. The ages of the persons employed in a certain office are as follows—3 aged 20, 1 aged 22, 2 aged 23, 3 aged 30, 1 aged 33, 4 aged 38, and 2 aged 47; what is the average age of the persons in that office?

The sum of their ages =

$$20 \times 3 + 22 + 23 \times 2 + 30 \times 3 + 33 + 38 \times 4 + 47 \times 2$$
$$= 60 + 22 + 46 + 90 + 33 + 152 + 94 = 497.$$

The number of persons in the office $= 3 + 1 + 2 + 3 + 1 + 4 + 2 = 16$.
The average age $\therefore \times \frac{497}{16} = 31\cdot0625$. *Ans.* 31·0625.

1858, £327 9*s.* 5*d.*, in 1859, £400, and 1860, £350 17*s.* 1*d.*; what was his average annual income?

Adding the different sums together we have for the total sum received in 5 years, £1680 15*s.*, and dividing this by 5 we get the average required, £336 3*s.* *Ans.* £336 3*s.*

Ex. 4. Out of 120 prisoners in a gaol, 63 can read and write, 21 more can read but not write; what is the percentage (1) of those who can read and write, (2) of those who can read?

This is equivalent to finding what number bears the same ratio to 100, (1) which 63 bears to 120, (2) which 84 bears to 120;

$$\therefore 120 : 100 :: 63 \qquad \tfrac{105}{2} = 52{\cdot}5$$
$$120 : 100 :: 84 \qquad 70$$

Ans. (1) 52·5. (2) 70.

Ex. 5. In 1841 the population of a town was 11933, and in 1861 it was 34621. Find the increase per cent. in the 20 years.

Subtracting 11933 from 34621 we obtain 22688, the increase in 20 years, and the question becomes, if 11933 give an increase of 22688 what increase will 100 give?

$$\therefore 11933 : 100 :: 22688,$$

which gives the required percentage, 190·12. *Ans.* 190·12.

Ex. 6. The population of a town, after losing 30 per cent. of its inhabitants by cholera, was 12075. What was its population before the outbreak?

There are 70 left out of every 100;

$$\therefore 70 : 12075 :: 100.$$ *Ans.* 17250.

Ex. 7. If, instead of a lb. weight, a tradesman uses one which weighs $15\frac{3}{4}$ oz., how much per cent. will he gain?

He will gain $\frac{1}{4}$ oz. in every $15\frac{3}{4}$ oz.; we have

$$\therefore 15\tfrac{3}{4} : 100 :: \tfrac{1}{4}.$$ *Ans.* $1\frac{37}{63}$ per cent.

Ex. 8. A spirit merchant buys 15 gallons of gin, at 14*s.* 6*d.* a gallon; how much water must he put to it so that he may gain £1 14*s.* 6*d.* by selling it at 14*s.* a gallon?

The gin costs him £10 17*s.* 6*d.*, so that to gain £1 14*s.* 6*d.* he must sell it for £12 12*s.*, or 252*s.* He must therefore sell $\frac{252}{14}$ gal. = 18 gallons, i.e. he must add 3 gallons of water. *Ans.* 3 gallons.

Ex. 9. A. can do a piece of work in 3 days, B. can do 3 times as much in 8 days, and C. five times as much in 12 days; in what time will they do it together?

A. can do $\frac{1}{3}$rd of the work in 1 day
B. „ $\frac{3}{8}$ths „ „
C. „ $\frac{5}{12}$ths „ „

∴ working altogether, they can do $\frac{1}{3}+\frac{3}{8}+\frac{5}{12}=\frac{9}{8}$ths of the work in on day. The time they will take to finish it will ∴ be $1\div\frac{9}{8}=\frac{8}{9}$ths of day. *Ans.* $\frac{8}{9}$ day.

Ex. 10. A cistern can be filled by 2 pipes, A. and B., in 20 min. an 24 min. respectively, and emptied by C. in 15 min. A. is open fo 4 min., when B. is also opened, and after 6 min. more C. is opened how long will the cistern be filling?

$\frac{1}{20}$ flows through A. in 1 min., $\frac{1}{24}$ through B., and $\frac{1}{15}$ through C. A. i open for 10 min. and B. for 6 min., ∴ the quantity of water in the cister $=\frac{10}{20}+\frac{6}{24}=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$. Hence $\frac{1}{4}$th more is required; but C. is now opened, and therefore the increase in the cistern per min. $=\frac{1}{20}+\frac{1}{24}-\frac{1}{15}=\frac{1}{40}$, and the time required to fill it $\frac{1}{4}\div\frac{1}{40}=10$ min., and therefore the time from A. being opened is 20 min. *Ans.* 20 min.

Ex. 11. A. can do a piece of work in 12 hours, B. in 4 hours, and C. in 3 hours. A., B., and C. all work together for half an hour, when A. leaves off. How long will it take B. and C. to finish the work?

A. can do $\frac{1}{12}$th in 1 hour
B. „ $\frac{1}{4}$th „
C. „ $\frac{1}{3}$rd „
∴ in half an hour A., B., and C. together can do $\frac{1}{2}$ of $(\frac{1}{12}+\frac{1}{4}+\frac{1}{3})=\frac{1}{3}$rd of the whole.

B. and C. have ∴ $\frac{2}{3}$rds to do, and their rate of work is $\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$ in an hour. They will ∴ be $\frac{2}{3}\div\frac{7}{12}=1\frac{1}{7}$ hours completing it.

Ans. $1\frac{1}{7}$ hour.

Ex. 12. A. and B. can do a piece of work in 10 days, B. and C. in 12 days, and A. and C. in 16 days; in what time can they do it separately?

A. and B. can do $\frac{1}{10}$th and B. and C $\frac{1}{12}$ in 1 day, ∴ B. can do $\frac{1}{10}-\frac{1}{12}=\frac{1}{60}$ *more than* C. in 1 day, and ∴ the work which B. and C. can do is $\frac{1}{60}$ of the whole more than double what C. can do; i. e. C. can do $(\frac{1}{16}-\frac{1}{60})\div 2=\frac{11}{480}$ths in one day, and would ∴ complete the work in $1\div\frac{11}{480}=\frac{480}{71}=43\frac{7}{11}$ days. Hence A. can do $\frac{1}{16}-\frac{11}{480}=\frac{19}{480}$ths in one day, and would ∴ complete it in $1\div\frac{19}{480}=\frac{480}{19}=25\frac{5}{19}$ days. Also B. can do $\frac{1}{12}-\frac{11}{480}=\frac{29}{480}$ths in one day, and would ∴ complete it in $1\div\frac{29}{480}=\frac{480}{29}=$ $16\frac{16}{29}$ days.

A. $25\frac{5}{19}$ days
B. $16\frac{16}{29}$ „
C. $43\frac{7}{11}$ „
Ans.

Ex. 13. A person rows $1\frac{1}{4}$ miles down stream in 20 minutes; i there had been no stream it would have taken him half an hour; wha return against it?

His rate per hour with the stream $= 1\frac{1}{2}$ m. $\times 3 = 4\frac{1}{2}$ m.
" " without " $= 1\frac{1}{2}$ m. $\times 2 = 3$ m.
$\therefore$ the rate of the stream $= 4\frac{1}{2} - 3 = 1\frac{1}{2}$ miles per hour.
His rate per hour *against* the stream $= 3 - 1\frac{1}{2} = 1\frac{1}{2}$ m.
$\therefore$ his time in returning $= 1\frac{1}{2} \div 1\frac{1}{2} = 1$ hour.

Ans. $1\frac{1}{2}$ miles per hour. 1 hour.

Ex. 14. Divide £1005 among A., B., and C. in such a manner that for every £4 A. receives, B. receives £5, and C. £6.

The number of pounds to be taken each time $= 4 + 5 + 6 = 15$, $\therefore$ the number of divisions $= \frac{1005}{15} = 67$, and at each division A. takes £4, B. £5, and C. £6; their respective shares are $\therefore$ 67×4, 67×5, and 67×6, or £268, £335, and £402.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

Find the G. C. M. of

(1) 945 and 1485; 6156 and 8721.
(2) 1274 and 1417; 1369 and 1776.
(3) 7623 and 7920; 10029 and 10609.
(4) 4429 and 4644; 37577 and 38637.
(5) 18648 and 21312; 114342 and 121363.

Find the L. C. M. of

(6) 6, 8, 16, 24, 30.
(7) 5, 9, 12, 15, 32, 36.
(8) 3, 4, 6, 8, 9, 12.
(9) 6, 7, 18, 28, 40, 42.
(10) 12, 20, 24, 54, 81, 63, 14.

VULGAR FRACTIONS.

(11) Multiply $\frac{7}{24}$ separately by 4, 5, 6, 7, 8, and 11.
(12) Multiply $\frac{4}{39}$ separately by 3, 5, 10, 13, and 14.
(13) Divide $\frac{12}{13}$ separately by 3, 4, 5, 6, and 7.
(14) Divide $\frac{18}{19}$ separately by 6, 9, 10, 11, and 18.
(15) Express 7 as a fraction with the denominators 2, 3, 4, 5, and 6.
(16) Express 8 as a fraction with the denominators 7, 8, 9, 10, and 11

Express as improper fractions

(17) $5\frac{3}{7}$, $10\frac{5}{13}$, $11\frac{21}{40}$.
(18) $12\frac{13}{20}$, $4\frac{11}{13}$, $17\frac{5}{27}$.
(19) $9\frac{56}{83}$, $20\frac{117}{119}$, $101\frac{100}{101}$.

Express as mixed numbers

(20) $\frac{37}{8}$, $\frac{23}{6}$, $\frac{49}{16}$.
(21) $\frac{137}{12}$, $\frac{157}{13}$, $\frac{193}{17}$.
(22) $\frac{2008}{155}$, $\frac{304}{117}$, $\frac{10021001}{1001}$.

Reduce to their lowest terms

(23) $\frac{324}{396}$, $\frac{315}{378}$, $\frac{1452}{2178}$.
(24) $\frac{108}{972}$, $\frac{7040}{7392}$, $\frac{3432}{2352}$.
(25) $\frac{1296}{1512}$, $\frac{2805}{3135}$, $\frac{1764}{2352}$.
(26) $\frac{3042}{3094}$, $\frac{6398}{7312}$, $\frac{69615}{92872}$.
(27) $\frac{21168}{204624}$, $\frac{9809}{11540}$, $\frac{2457}{3276}$.
(28) $\frac{23779}{25585}$, $\frac{95469}{359784}$, $\frac{527751}{645029}$.
(29) $\frac{1099901}{1067803}$, $\frac{23205}{46436}$, $\frac{81103}{209979}$.

Express as simple fractions

(30) $\frac{1}{6}$ of $\frac{4}{5}$ of $\frac{15}{28}$ of 7; $\frac{5}{9}$ of $\frac{22}{25}$ of $\frac{9}{11}$.
(31) $\frac{3}{5}$ of $\frac{5}{6}$ of 8; $\frac{2}{3}$ of $\frac{7}{13}$ of $\frac{9}{35}$ of $\frac{5}{12}$ of 26.
(32) $3\frac{1}{2}$ of $4\frac{5}{7}$ of $1\frac{1}{11}$ of $2\frac{1}{4}$; $\frac{12}{13}$ of $2\frac{1}{3}$ of $1\frac{1}{25}$ of $1\frac{11}{64}$.
(33) $\frac{5}{18}$ of $\frac{7}{3}$ of $3\frac{3}{5}$ of $2\frac{1}{4}$ of $\frac{3}{10}$ of 2 of $\frac{8}{27}$.
(34) $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{6}{7}$ of $70\frac{2}{9}$ of $\frac{3}{40}$ of $1\frac{7}{11}$ of 147.

Reduce to equivalent fractions with the least common denominator.

(35) $\frac{7}{36}$, $\frac{5}{8}$, $\frac{8}{9}$, $\frac{43}{48}$, $\frac{15}{16}$; $\frac{4}{7}$, $\frac{11}{15}$, $\frac{23}{24}$, $\frac{37}{35}$, $\frac{9}{42}$.
(36) $\frac{17}{18}$, $\frac{5}{12}$, $\frac{25}{102}$, $\frac{8}{63}$, $\frac{7}{16}$; $\frac{5}{9}$, $\frac{10}{17}$, $\frac{21}{68}$, $\frac{13}{126}$, $\frac{5}{7}$.

(37) $\frac{3}{4}, \frac{5}{6}, \frac{7}{9}, \frac{5}{12}; \frac{1}{3}, \frac{3}{5}, \frac{7}{10}, \frac{9}{14}$.

(38) $\frac{1}{4}, \frac{5}{12}, \frac{7}{15}, \frac{3}{20}; \frac{1}{2}, \frac{7}{12}, \frac{5}{6}, \frac{4}{5}$.

(39) $5\frac{4}{9}, \frac{3}{8}, \frac{7}{12}, 2\frac{5}{24}; 7\frac{1}{8}, \frac{2}{3}, 3\frac{4}{5}, 1\frac{11}{12}$.

(40) $12\frac{2}{3}, 2\frac{11}{16}, \frac{23}{24}; \frac{11}{12}, 5\frac{13}{15}, \frac{21}{20}, 4\frac{5}{18}$.

(41) $8\frac{1}{7}, \frac{9}{14}, 4\frac{5}{21}, 2\frac{17}{42}$ $12\frac{1}{9}, \frac{7}{15}, \frac{47}{20}, \frac{7}{45}$.

(42) $\frac{2}{5}, 2\frac{5}{6}, 2\frac{1}{3}, \frac{5}{12}; \frac{4}{5}, \frac{7}{8}, \frac{11}{12}, 3\frac{3}{20}$.

(43) $7\frac{1}{3}, \frac{3}{14}, \frac{10}{21}, 2\frac{7}{9}; 1\frac{2}{5}, 2\frac{3}{7}, \frac{3}{13}$ of $7\frac{4}{5}$.

(44) $\frac{3}{4}, 5\frac{2}{3}, 1\frac{7}{12}, \frac{5}{96}, 3\frac{11}{100}; \frac{7}{18}, 6\frac{5}{48}, \frac{17}{8}, \frac{13}{24}$.

(45) $\frac{5}{6}$ of $\frac{12}{15} + \frac{1}{5} + \frac{2}{15} + 1\frac{4}{21}$ of $1\frac{1}{5} + \frac{4}{105}$.

(46) $2\frac{1}{3}$ of $1\frac{1}{14} + 1\frac{5}{12} + 3\frac{3}{5}$ of $1\frac{2}{9}$ of $1\frac{4}{11} + 2\frac{2}{13}$ of $5\frac{4}{7}$.

Subtract

(47) $\frac{3}{4}$ from $\frac{7}{8}$; $\frac{3}{8}$ from $\frac{11}{12}$; $2\frac{7}{18}$; from $3\frac{7}{9}$.

(48) $\frac{7}{12}$ from $1\frac{17}{24}$; $5\frac{6}{35}$ from $7\frac{4}{5}$; $2\frac{5}{9}$ from $3\frac{3}{7}$.

(49) $6\frac{3}{8}$ from $17\frac{1}{4}$; $19\frac{7}{9}$ from $32\frac{3}{4}$; $1\frac{7}{9}$ from $16\frac{2}{15}$.

(50) $1\frac{7}{9}$ from $5\frac{3}{4}$; $\frac{29}{60}$ from $1\frac{4}{15}$; $5\frac{11}{16}$ from $8\frac{11}{24}$.

(51) $3\frac{4}{45}$ from $8\frac{1}{30}$; $\frac{19}{52}$ from $4\frac{3}{26}$; $7\frac{15}{22}$ from $11\frac{3}{44}$.

(52) The sum of $\frac{1}{3} + \frac{2}{7} + \frac{5}{21} + 1\frac{2}{11} + \frac{8}{77}$ from $2\frac{1}{6}$.

Find the value of

(53) $3\frac{5}{6} - 1\frac{7}{12} + 1\frac{4}{9} - 2\frac{11}{72}$.

(54) $7\frac{9}{10} + 3\frac{2}{3} - 6\frac{3}{8} + 5\frac{3}{5} - 2\frac{2}{3}$ of $1\frac{5}{16}$.

(55) $2\frac{3}{7}$ of $5\frac{5}{6} + 1\frac{11}{12} - 3\frac{5}{8}$ of $1\frac{1}{29} + 1\frac{5}{8} - 2\frac{2}{5}$ of $\frac{5}{6}$.

(56) $3\frac{2}{3} - 6\frac{1}{5}$ of $3\frac{3}{7}$ of $\frac{1}{14} - \frac{7}{9}$ of $6\frac{3}{10} + 3\frac{11}{20}$.

(57) $3\frac{5}{9}$ of $1\frac{5}{16}$ of $1\frac{2}{7} - 5\frac{3}{4}$ of $1\frac{7}{23} + 3\frac{7}{8} - 2\frac{2}{5}$ of $3\frac{1}{3}$ of $\frac{1}{5}$.

Find the product of

(58) $\frac{7}{9} \times \frac{3}{4} \times \frac{15}{28}; \frac{5}{6} \times \frac{3}{13} \times \frac{52}{55}$.

(59) $42\frac{3}{5} \times \frac{10}{27}; 19\frac{5}{6} \times 3\frac{3}{17}; 9\frac{4}{5} \times 5\frac{5}{7}$.

(60) $161\frac{7}{8} \times 4\frac{4}{5}; 22\frac{2}{11} \times 4\frac{2}{5}; 15\frac{5}{9} \times 5\frac{1}{7} \times 1\frac{1}{20}$.

(61) $9\frac{5}{11} \times 4\frac{1}{8}; 18\frac{3}{4} \times 1\frac{13}{15}; \frac{19}{20} \times 11\frac{1}{9} \times 3\frac{21}{88}$.

(62) $\frac{21}{26} \times \frac{20}{51}; 20\frac{4}{9} \times 1\frac{13}{23}; 38\frac{1}{4} \times 2\frac{2}{17}$.

Divide

(63) $\frac{7}{18}$ by $\frac{4}{9}$; $1\frac{6}{11}$ by $3\frac{1}{11}$; $8\frac{4}{5}$ by $6\frac{2}{7}$.

(64) $15\frac{7}{12}$ by $7\frac{5}{9}$; $4\frac{5}{6}$ by $6\frac{7}{18}$; $7\frac{5}{7}$ by $4\frac{2}{21}$.

(65) $2\frac{4}{15}$ by $4\frac{1}{3}$; $19\frac{2}{7}$ by $4\frac{3}{8}$; $\frac{24}{35}$ by $3\frac{3}{7}$.

(66) $72\frac{1}{2}$ by $7\frac{1}{4}$; $1\frac{7}{22}$ by $8\frac{2}{7}$; $7\frac{7}{11}$ by $3\frac{3}{55}$.

(67) $\frac{51}{120}$ by $\frac{7}{18}$; $2\frac{1}{12}$ by $4\frac{1}{4}$; $11\frac{3}{5}$ by $2\frac{9}{10}$.

(68) $\dfrac{2\frac{4}{11}}{2\frac{3}{5}}$ by $\dfrac{2\frac{7}{11}}{8\frac{7}{10}}$; $\dfrac{2\frac{2}{3}}{2\frac{2}{15}}$ by $1\frac{1}{4}$.

(69) $(\frac{1}{3}$ of $\frac{1}{2} + \frac{3}{2}$ of $5)$ by $(9\frac{1}{3} - 1\frac{2}{3})$.

(70) $(\frac{2}{11}$ of $6\frac{10}{13}$ of $14\frac{3}{10})$ by $(\frac{1}{17}$ of $299\frac{1}{5})$.

(71) $(\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5})$ by $(\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5})$.

(72) $(4\frac{4}{5}-2\frac{5}{8}+1\frac{3}{20}-2\frac{3}{4})$ by $(3\frac{4}{5}+2\frac{5}{8}-1\frac{1}{10}+4\frac{3}{4})$.

(73) $\frac{3}{5}+\frac{4}{9}+\frac{3}{10}+\frac{7}{18}$ by $7\frac{5}{6}$.

(74) Divide the sum of $\frac{1}{6}$, $2\frac{1}{7}$, and $13\frac{3}{10}$ by $13\frac{1}{14}$, and subtract the result from $5\frac{3}{70}$.

(75) Subtract the sum of $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{96}$, $\frac{11}{100}$ from $2\frac{1}{6}$.

(76) Multiply $\frac{3}{5}$ of $\frac{7}{11}$ of 15 by $\frac{14}{15}$ of $12\frac{4}{7}$.

(77) Reduce $(\frac{1}{7}$ of $2\frac{1}{3}$ of $6\frac{7}{8})\div 4\frac{2}{5}$ to a simple fraction, and multiply the result by $1\frac{23}{25}$.

(78) Reduce $(\frac{1}{8}$ of $2\frac{2}{3}$ of $15\frac{3}{5})\div 16\frac{5}{7}$ to a simple fraction, and multiply the result by $6\frac{3}{7}$.

(79) Subtract $\dfrac{3\frac{4}{5}+\frac{29}{45}}{3\frac{3}{4}+14\frac{1}{36}}$ from $\dfrac{7\frac{1}{2}+1\frac{17}{22}}{8\frac{2}{3}+3\frac{23}{33}}$.

(80) Divide $\dfrac{3\frac{1}{2}-2\frac{1}{6}}{\frac{1}{4}\text{ of }(\frac{1}{5}+\frac{1}{7})}$ by $15\frac{5}{9}$.

Simplify

(81) $\frac{3}{5}+1\frac{1}{3}+\frac{7}{6}+\frac{11}{30}+\frac{2}{45}-\frac{7}{18}$.

(82) $\frac{1}{7}+\frac{3}{11}+\frac{17}{13}-\frac{581}{1001}$.

(83) $\frac{2}{3}$ of $3\frac{3}{10}+\dfrac{1\frac{3}{4}}{2\frac{5}{8}}$ of $17+\frac{3}{5}$ of $5\frac{3}{4}$ of $\frac{34}{51}$.

(84) $\frac{3}{7}$ of $\dfrac{4\frac{5}{9}}{12\frac{1}{3}}$ of $\dfrac{3\frac{4}{11}}{11\frac{5}{7}}\div 1\frac{1}{11}$.

(85) $\frac{2}{7}$ of $\dfrac{\frac{5}{11}\text{ of }5\frac{1}{2}\div 2\frac{1}{3}}{\frac{4}{9}\text{ of }\frac{3}{13}\div 8\frac{2}{3}}$.

(86) $\dfrac{1\frac{3}{4}-\frac{7}{8}\text{ of }\frac{9}{14}}{\frac{5}{6}\text{ of }\frac{6}{13}+5\frac{1}{2}}$.

(87) $\dfrac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}+\frac{7}{12}$ of $\dfrac{9\times 10}{14\times 13}-\dfrac{22\frac{1}{2}}{30}$.

(88) $\dfrac{5\frac{1}{3}-2\frac{7}{8}+3\frac{1}{2}}{5\frac{1}{3}+2\frac{7}{8}-3\frac{1}{2}}\div 1\frac{30}{113}$.

(89) $\frac{7}{8}$ of $\dfrac{2\frac{3}{4}+3\frac{3}{5}}{4\frac{1}{5}+5\frac{1}{4}}+\dfrac{3\frac{2}{3}}{10\frac{1}{2}}$.

(90) $\dfrac{2\frac{1}{4}+3\frac{1}{28}}{3\frac{16}{35}+12\frac{3}{5}}+\dfrac{6\frac{2}{3}+3\frac{19}{39}}{12\frac{19}{26}+2\frac{1}{2}}$.

(91) $\frac{15}{17}$ of $\dfrac{6\frac{4}{5}}{2\frac{8}{11}}$ of $(2\frac{7}{9}\div 3\frac{8}{9})$.

(92) $(\frac{1}{2}-\frac{3}{10}+\frac{1}{7}+\frac{2}{35})\times(4\frac{2}{7}+\frac{9}{8}-\frac{5}{14}-\frac{3}{56})$.

(93) $\left(\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{3}\times 3\frac{1}{3}+\frac{13}{36}}-\dfrac{1}{2\frac{1}{2}}\right)\div\dfrac{1}{1\frac{2}{3}}$.

(94) $9\frac{1}{3}\div\dfrac{5\frac{1}{2}+6\frac{3}{7}-10\frac{5}{28}}{4\frac{1}{3}-7\frac{5}{6}+3\frac{7}{8}}$.

(95) $\dfrac{6\frac{4}{5}-3\frac{3}{5}}{6\frac{4}{5}+3\frac{3}{5}}$ of $\dfrac{8\frac{3}{4}-4\frac{11}{12}}{3\frac{7}{13}}$ of $\dfrac{3\frac{7}{8}+2\frac{1}{2}}{9\frac{5}{6}-3\frac{1}{3}}$.

(96) $\dfrac{\frac{2}{3}+\frac{3}{4}+\frac{4}{5}}{\frac{4}{5}+\frac{5}{7}+\frac{7}{9}}\times\dfrac{\frac{4}{5}\text{ of }\frac{5}{7}\text{ of }\frac{7}{9}}{\frac{2}{3}\text{ of }\frac{3}{4}\text{ of }\frac{4}{5}}$.

(97) $(2\frac{15}{17}+3\frac{1}{16})\times(2\frac{3}{7}-1\frac{6}{11})\div\dfrac{\frac{2}{11}+\frac{1}{5}}{1\frac{3}{11}-1\frac{1}{5}}$.

(98) $\left\{\dfrac{4\frac{1}{4}-3\frac{3}{8}}{4\frac{1}{4}+3\frac{3}{8}}+\dfrac{3-2\frac{1}{3}}{4-3\frac{1}{4}}\right\}\div\left\{\dfrac{1}{3\frac{1}{5}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{5}{8}}{2}+\dfrac{\frac{4}{7}}{4\frac{4}{7}}\right\}$.

(99) What number added to $\dfrac{5\frac{6}{7}-6\frac{1}{2}+2\frac{4}{5}}{16\frac{2}{3}-5\frac{5}{7}-10\frac{9}{10}}$ will make 45?

(100) To what number must $\dfrac{1\frac{2}{3}+1\frac{5}{8}-4\frac{3}{4}-2\frac{12}{13}}{\frac{2}{3}-2\frac{5}{8}+4\frac{3}{4}-2\frac{12}{13}}$ be added to make $\frac{35}{41}$?

(101) What number multiplied by $5\frac{5}{6}$ of $1\frac{5}{7}-\frac{2}{3}$ will give $3\frac{1}{2}$?

$$\frac{1+\frac{1}{2}}{2+\frac{1}{3}} \text{ and } \cfrac{1+\frac{2}{3}}{\cfrac{1+\frac{3}{4}}{1+\frac{4}{5}}}.$$

(104) If I pay away $\frac{1}{5}$ of my money, then $\frac{1}{2}$ of what remains, and ien $\frac{1}{3}$ of what still remains; what fraction of the whole shall I have ft?

(105) $\frac{3}{7}$ths of a ship belongs to A., he sells $\frac{2}{3}$rds of his share to B.; hat part of the ship belongs to him now?

DECIMAL FRACTIONS.

Add together

(106) 3·046, ·0212, 47·9, ·6, and ·0003.
(107) 52·38, 367·4, ·172, 6·0053, and 3.
(108) 672·5, 4·923, 80, ·076, and 9·1.
(109) 1·4693, ·001, ·6, 6, 321·2, and 5·09.
(110) 42·79, ·2105, ·047, 140, ·000001, and 78·98007.
(111) ·813, 420·91, ·00093, 7·043, and 12560.
(112) 50004, ·46, ·579, and 1201·043.
(113) ·2764, 1824, 176, ·09032, and ·47.
(114) 802·573, 31·095, 9·7235, and ·507.
(115) ·024, 1000, ·00565, and 2·432.

Subtract

(116) 16·498 from 503·12; 19·7564 from 26.
(117) 236·932 from 270·00086; 3·901 from 6·01.
(118) 42·946 from 161·06; 83·70081 from 84·001.
(119) 5·790007 from 10·95003; 27·8 from 49·30101.
(120) 17·5986 from 19· 987; 156·95078 from 200.
(121) 59·87 from 506·222; 19·9876 from 75.
(122) 7·2014 from 872·70032; 11·709 from 12·0954.
(123) ·099095 from 1·007; ·8309 from 407·02.
(124) What number added to 10·0001 will make 11?
(125) What number subtracted from 10·0001 will leave 9?

Multiply

(126) 32·5 by ·0763; 9·436 by 67·49.
(127) 4·81 by ·0074; 6·4073 by ·42.
(128) 65·43 by ·00376; 192·336 by ·73307.
(129) 5412·384 by 1·0023; 1·02 by 3·067.
(130) 757·04 by 15·8; 45·267 by 7·045.
(131) 7·045 by 974·01; 34·742 by 3·25.

(132) ·51904 by 60·504; 1703·02 by 93·05.

(133) 95·376 by ·0283; 2·7 by 27, and the product by ·027.

(134) 5·01 by ·0001, the product by 33·009, and subtract the result from 2001.

Divide

(135) 1735·5 by 6·5; 1708·4592 by ·0072.

(136) ·01 by ·00001; ·1872 by ·0312.

(137) 15·6 by ·12, by 1·2, and by 12.

(138) 18·016 by ·2, and the quotient by ·0002

(139) 15·96 by ·065; 24·1 by ·0025.

(140) ·0078125 by ·00125; ·000123123 by ·0032.

(141) ·0088266 by ·000235; 17·28 by ·0144.

(142) ·016 by ·2 and the quotient by ·0002; 21546·872 by ·0542.

(143) 22·097 by 543·14 to 5 places of decimals.

(144) 169·24 by 549 to 3 places of decimals.

(145) 112·03 by 98·52 to 3 places of decimals.

(146) 1·765 by 4470 to 5 places of decimals.

(147) 169 by ·13, by 1300, and by ·013; ·121 by 11, by 110, and by ·0011.

(148) What number divided by 1·25 will give ·005 for the quotient?

(149) Find the difference between 13·2 divided by 11 and by ·11.

(150) Find the sum, difference, and product of ·000182 divided by 1·3 and divided by 1·4.

(151) Reduce to its simplest value $\frac{435{\cdot}1 \times {\cdot}0046}{{\cdot}12500}$.

(152) The Julian year contains 365·25 days, while the true solar year contains 365·242218 days; what will be the error in 12 centuries from using the Julian year?

(153) Find the diameter of the earth, supposing the sun's diameter, which is 111·454 times as great, to be 883345 miles?

Reduce to decimals

(154) $\frac{1}{4}, \frac{3}{8}, \frac{1}{400}$. (155) $\frac{11}{16}, \frac{29}{3200}, \frac{5}{64}$. (156) $\frac{77}{128}, \frac{137}{625}, \frac{21}{250}$.

(157) $\frac{14}{5}, \frac{157}{1250}, 13\frac{17}{50}, (\frac{3}{4})^2$. (158) $\frac{2}{3}, \frac{6}{7}, \frac{10}{11}$. (159) $\frac{1}{13}, \frac{11}{15}, \frac{23}{33}$.

(160) $\frac{11}{21}, \frac{36}{275}, \frac{1}{148}$. (161) $\frac{243}{6075}, \frac{729}{12150}, \frac{241}{15424}$.

(162) $(\frac{9}{11} - \frac{3}{16}) \times (5\frac{1}{8} + 2\frac{7}{15}) \times (3\frac{5}{6} - \frac{11}{12})$ to 5 places of decimals.

(163) $\frac{1\frac{3}{4} - \frac{7}{8} \text{ of } \frac{9}{14}}{5\frac{1}{2} - (\frac{5}{6} \text{ of } \frac{6}{13})}$; $(7\frac{1}{2} \text{ of } \frac{1}{25}) + \frac{17}{25}$.

Reduce to vulgar fractions in their lowest terms

(164) ·01056; 23·038125; ·0006875.

(167) $3\cdot0724$; $\cdot025$; 097824. (168) $\cdot\dot{3}$; $\cdot0\dot{3}\dot{3}$; $\cdot7\dot{2}4\dot{5}$.

(169) $\cdot158\dot{6}\dot{3}$; $\cdot05\dot{1}$; $\cdot1\dot{3}\dot{5}$. (170) $\cdot012\dot{3}\dot{6}$; $\cdot1\dot{4}$; $342\dot{1}\dot{8}$.

(171) $11\cdot2\dot{8}\dot{7}$; $\cdot13\dot{0}\dot{9}$; $\cdot3782\dot{1}\dot{6}$. (172) $\cdot6012\dot{4}\dot{3}$; $\cdot2\dot{6}1538\dot{4}$; $9\cdot92\dot{8}$.

Find the value correct to 5 places of decimals.

(173) $3\cdot\dot{7}5\dot{5} + 7 + \cdot05\dot{8} + \cdot\dot{0}03\dot{6}$.

(174) $25\cdot4\dot{9}6\dot{3} + \dot{0}\dot{9} + 13\cdot5\dot{6}2\dot{7} + \cdot78\dot{5}$.

(175) $93\cdot9865\dot{4}\dot{3} + 10\cdot\dot{0}70\dot{7} + \cdot0005 + 1$.

(176) $\cdot\dot{7} + 6\cdot9\dot{4} + \cdot\dot{0}2\dot{5} + 6\cdot78\dot{9}\dot{8} + 3\cdot7\dot{8}6\dot{5}$.

(177) $2\cdot\dot{7} - \cdot9\dot{1}\dot{3}$; $5 - 3\cdot95\dot{7}8\dot{6}$.

(178) $17\cdot856\dot{4} - \cdot08\dot{7}$; $3\cdot5\dot{9}8\dot{7} - 2\cdot\dot{6}$.

Find the value of

(179) $\cdot1\dot{3} \times 2\cdot3\dot{4}\dot{5}$; $\cdot0\dot{6} \times \cdot18\dot{7}$; $5\cdot\dot{0}9\dot{7} \times 12$.

(180) $\cdot\dot{5}2380\dot{9} \times 1\cdot\dot{7}\dot{2}$; $\cdot\dot{5}3846\dot{1} \times 2\cdot\dot{7}85714\dot{2}$; $\cdot8\dot{3} \times 1\cdot\dot{6}\dot{3}$.

(181) $\cdot\dot{3} \div \cdot\dot{0}\dot{9}$; $042 \div \cdot03\dot{6}$.

(182) $\cdot\dot{0}\dot{4} \div \cdot\dot{7}6923\dot{0}$; $61\cdot\dot{1}6\dot{5} \div 5\cdot\dot{0}9\dot{7}$.

(183) $\cdot\dot{3} \div \cdot\dot{2}$; $\cdot0\dot{0}2\dot{1} \div 48\cdot02\dot{6}$.

(184) $4\cdot03 \div \cdot1\dot{2}1\dot{0}$; $42\cdot04\dot{6}\dot{3} \div \cdot\dot{1}3\dot{6}$.

(185) $(12\cdot5 + 1\cdot25) \div (12\cdot5 - 1\cdot25)$.

(186) $(3\frac{1}{2} - \cdot04) \div (5 - \cdot0625)$.

(187) $\frac{3}{5} + \cdot14 + \frac{3}{4}$ of $1\cdot0784$.

(188) $10\frac{3}{8} + 1\cdot125 + \frac{7}{10} + \frac{13}{16}$.

(189) Reduce $\frac{1}{16} + \frac{9}{400} - \frac{13}{625}$ to a decimal.

(190) Simplify, expressing the result, (1) as a fraction, (2) as a decimal

$$\frac{\cdot5 + \frac{1}{4} + \cdot1 + \frac{1}{8} + \cdot125}{\frac{1}{3} + \cdot2 + \frac{1}{9} + \frac{1}{15}}.$$

(191) Show that $\dfrac{46\cdot2}{92\cdot4} = \cdot75 \times \cdot\dot{6}$.

(192) Express $\dfrac{48\frac{4}{7}}{1085\frac{7}{10}} \div \dfrac{7\frac{3}{11}}{174\frac{3}{17}}$ as a decimal.

(193) What number must be subtracted from $\dfrac{2\frac{2}{15} \times 2\frac{5}{8}}{2\frac{3}{5} - 2\frac{1}{4}}$ to leave $4\cdot24317$?

(194) Divide 356400 by $\cdot66$; and $\cdot3564$ by 660.

(195) Find the value of $\dfrac{1\cdot1\dot{3}}{\cdot00\dot{0}13\dot{2}}$.

(196) Divide $12\cdot55$ by $\cdot01004$, 1255 by $10\cdot04$, and $\cdot001255$ by 1004.

(197) Add together $1\frac{1}{12}$, $3\frac{2}{15}$, $3\frac{3}{20}$, $4\frac{4}{27}$. (1) as fractions, (2) as decimals, and show that the results are the same.

(198) Express as a decimal $(\frac{9}{11}$ of $35\frac{1}{5} - 3\frac{1}{8}) + (2\cdot5625 + 7\frac{1}{4})$.

(199) Reduce $(\frac{3}{50}$ of $11\cdot8 - \frac{5}{80}$ of $11\cdot02) \div \cdot1$ to a decimal.

(200) Reduce $\dfrac{4\frac{5}{6} + 5\frac{8}{7} - 9\frac{7}{9} + 3\frac{101}{126}}{3\frac{1}{2} + 5\frac{1}{7} - 4\frac{9}{14}}$ to a decimal.

REDUCTION.

Reduce

(201) £527 16*s*. $8\frac{1}{2}$*d*. to farthings; £393 11*s*. $7\frac{1}{2}$*d*. to halfp
(202) £9 19*s*. $11\frac{3}{4}$*d*. to farthings 719 gui. 15*s*. $4\frac{3}{4}$*d*. to fart
(203) 1827 moidores to pence 9753 halfcrowns to farthin
(204) 117 tons 3 qrs. to ounces.
(205) 49 tons 15 cwt. 2 qrs. 14 lbs. to pounds.
(206) 35 cwt. 27 lbs. 13 oz., 7 drs. to drams.
(207) 16 tons 4 cwt. 3 qrs. 15 lbs. 13 oz. to ounces.
(208) 5 tons 17 cwt. 1 qr. 11 oz. to drams.
(209) 12 cwt. 3 qrs. 13 lbs. 5 oz. to ounces.
(210) 17 tons 11 cwt. 1 qr. 8 lbs. 11 oz. to ounces.
(211) 18 tons 9 cwt. 3 qrs. 15 lbs. to ounces.
(212) 375 cwt. 2 qrs. 1 st. 13 lbs. to ounces.
(213) 47 cwt. 1 qr. 16 lbs. 2 oz. to drams.
(214) 63 cwt. 2 qrs. 5 oz. to drams.
(215) 54 cwt. 3 qrs. 18 lbs. 14 oz. to drams.
(216) 29 cwt. 1 qr. 11 drs. to drams.
(217) 171 tons 2 qrs. 15 lbs. 15 oz. 12 drs. to drams.
(218) 4 tons 17 cwt. 3 qrs. 11 lbs. 6 oz. to ounces.
(219) 17 lbs. 6 oz. 12 dwts. 7 grs. to grains.
(220) 151 lbs. 11 oz. 17 grs. to grains.
(221) 239 lbs. 7 dwts. to pennyweights.
(222) 399 lbs. 9 oz. 19 dwts. 21 grs. to grains.
(223) 3 lbs. 10 oz. 4 dwts. 12 grs. to grains.
(224) 17 lbs. 3 oz. 7 dwts. to grains.
(225) 15 lbs. 10 oz. 14 grs., troy, to grains.
(226) 13 lbs. 7 oz. 16 grs., troy, to grains.
(227) 14 lbs. 11 oz. 1 grs., troy, to grains.
(228) 46 lbs. 3 oz. 2 dwts. 2 grs. to grains.
(229) 13 lbs. 9 oz. 15 dwts. 13 grs. to grains.
(230) 23 oz. 2 scr. to scruples.
(231) 8 oz. 5 drs. 1 scr. 9 grs. to grains.
(232) 27 lbs. 10 oz. 7 drs. 17 grs. to grains.
(233) 55 oz. 4 drs. 2 scr. to grains.
(234) 11 m. 6 po. 4 yds. to feet.
(235) 13 m. 5 fur. 9 po. 3 yds. to inches.
(236) 42 m. 5 fur. 13 po. to inches.
(237) 23 m. 7 fur. 171 yds. 2 ft. to inches.
(238) 175 m. 3 fur. 15 po. 5 yds. 2 ft. 6 in. to inches.
(239) 31 lea. 6 fur. 37 po. 1 ft. to feet.

(242) 9 m. 3 fur. 6 po. 1 yd. to feet.
(243) 13 m. 5 fur. 171 yds. 2 ft. to inches.
(244) 5 m. 3 fur. 4 po. 3 yds. to feet.
(245) 13 s. yds. 5 s. ft. 19 s. in. to s. inches.
(246) 3 a. 20 p. 12 s. yds. 7 s. ft. to s. feet.
(247) 496 s. yds. 2 s. ft. to s. inches.
(248) 5 a. 13 p. to s. feet.
(249) 17 a. 3 r. 11 p. 27 s. yds. 5 s. ft. 57 s. in. to s. inches.
(250) 155 a. 21 s. yds. to s. feet.
(251) 15 a. 1 r. 17 p. 3 s. yds. 7 s. ft. to s. inches.
(252) 13 c. yds. 5 c. ft. 19 c. in. to cubic inches.
(253) 117 c. yds. 17 c. ft. 117 c. in. to cubic inches.
(254) 17 c. yds. 1001 c. in. to cubic inches.
(255) 17 qrs. 5 bus. 1 pk. 2 qts. to pints.
(256) 12 lds. 4 qrs. 7 bus. 3 pks. to gills.
(257) 159 quarters of corn to quarts.
(258) 467 tuns 3 hhds. 26 gals. 2 qts. 1 pt. of wine to pints.
(259) 7 weeks 3 days 17 hrs. 23 min. to seconds.
(260) 35 days 8 hrs. 48 min. 29 sec. to seconds.
(261) 210797 halfpence to pounds; 3719477 farthings to pounds.
(262) 7825963 farthings to guineas; 789573 pence to moidores.
(263) 375197 sixpences to guineas; 864528 groats to crowns.
(264) 983767 oz. to cwt.; 796835 drs. to quarters.
(265) 259566 lbs. to tons; 5346907 oz. to tons.
(266) 78904465 drs. to cwt.; 6857857 drs. to tons.
(267) 28726 grs. (troy to lbs.; 798765 dwts. to lbs.
(268) 900075 grs. (troy) to lbs.; 980706 grs. (troy) to lbs.
(269) In 237458 inches, how many miles, fur., &c.?
(270) In 5641080 inches, how many fur., po., &c.?
(271) In 982424 feet, how many miles, fur., po., &c.?
(272) In 870063 feet, how many fur., po., &c.
(273) In 5792685 inches, how many miles, fur., po., &c.?
(274) In 3126749 inches, how many miles, fur., &c.?
(275) In 200108 yards, how many leagues, miles, &c.?
(276) In 98769073 inches, how many miles, fur &c.?
(277) In 785900 inches, how many miles, fur., &c.?
(278) In 87053107 inches, how many poles, yds., &c.?
(279) In 569705 s. feet, how many acres, roods, &c.?
(280) In 1760075 s. feet, how many acres, roods, &c.?
(281) In 795014379 s. inches, how many roods, &c.?

(282) In 30007005 s. inches, how many acres, roods, &c.?

(283) In 917895601 s. inches, how many acres, roods, &c.?

(284) In seven hundred millions, seventy thousand, and seven s. inches, how many acres, roods, &c.?

(285) In 986877 c. inches, how many cubic yds., &c.?

(286) In 7968543 c. inches, how many cubic yds., &c.?

(287) In fifteen thousand and five millions, six hundred thousand, three hundred and nine c. inches, how many cubic yds., &c.?

(288) Reduce 9044 pints to quarters, and 133056 gills to loads.

(289) In 29336935 seconds, how many weeks, &c.?

(290) In 16900750337 minutes, how many months, &c.?

(291) In 2306090 ounces, how many tons, cwt., &c.?

(292) Reduce 600 half guineas to halfcrowns.

(293) How many lbs., oz., &c., are there in 721572 grains of gold?

(294) In 5864542 inches, how many miles, fur., &c.?

(295) In 2771443 seconds, how many weeks, days, &c.?

(296) In 8709354 inches, how many miles, fur., &c.?

(297) In 2160000 c. inches, how many loads of hewn timber?

(298) In 760320 inches, how many miles, fur., &c.?

(299) In 34677 s. yds., how many acres, roods, &c.?

(300) In 175983 c. inches, how many cubic yards, &c.?

(301) In 266450 grains of gold, how many lbs., &c.?

(302) Reduce 174283 grains (troy) to lbs.

(303) In 344612 ounces, how many cwts., &c.?

(304) In 787312 drams, how many tons, &c.?

(305) In 247391 inches, how many miles, fur., &c.?

(306) In 138297 c. inches, how many cubic yards, &c.?

(307) In 2099520 c. inches, how many loads of rough timber?

(308) In 1847638 inches, how many miles, fur., &c.?

(309) In 657918 c. inches, how many cubic yards, &c.

(310) How many francs, worth 9½*d.* each, are there in £693 14*s.* 9*d.*?

(311) How many moidores are there in 64800 farthings?

(312) How many half crowns are there in 83645 dollars, each worth 4*s.* 6*d.*?

(313) How many lbs. of silver are there in 270 spoons each weighing 1 oz. 13 dwts. 8 grs.?

(314) How many guineas are there in £567?

(315) How many times will a wheel, 3¼ yds. in circumference, turn round between London and York, a distance of 198 miles?

(316) How many tons, &c., of iron are there in 197 bars, each weighing 3 qrs. 8 lbs.?

many seconds are there in 25 years and 25 days?

(318) If a piece of waste land contain 24 acres, and 17 a. 3 r. be enclosed, how many perches are there in the remainder?

(319) Supposing a man to step 2 ft. 6 in., how many steps will he take in a mile?

(320) At the International Exhibition £2857 15*s.* was received in one day by shilling admissions; how many people must be admitted by payment on a half-crown day to make up the same amount?

COMPOUND ADDITION.

Add together

(321) £27 19*s.* 7¾*d.*, £9 0*s.* 6*d.*, £134 8*s.* 0½*d.*, £78 13*s.* 11¾*d.*, £1065 12*s.* 8¼*d.*, and £6976 7*s.* 5½*d.*

(322) £113 3*s.* 8¼*d.*, £36 14*s.* 7½*d.*, £7008 15*s.* 4*d.*, £5 19*s.* 11¾*d.*, £2647 8*s.* 6*d.*, and £79 13*s.* 9½*d.*

(323) £7 8*s.* 1½*d.*, £156 17*s.* 10*d.*, £43 4*s.* 7¾*d.*, £1068 12*s.* 4*d.* £975 19*s.* 5¾*d.*, and £7822 6*s.* 2½*d.*

(324) £3507 13*s.* 8*d.*, £6 14*s.* 7¼*d.*, 5*s.* 10½*d.*, £33 18*s.* 0¾*d.*, £144 17*s.* 5*d.*, and £27 4*s.* 10½*d.*

(325) £200 7*s.* 2*d.*, £41379 8*s.* 9½*d.*, 13*s.* 8¾*d.*, £46 0*s.* 7½*d.*, £1008 19*s.* 2¼*d.*, and £2 4*s.* 11*d.*

(326) £97 6*s.* 1*d.*, £168 17*s.* 4¼*d.*, £9 10*s.* 11*d.*, £7883 9*s.* 10¾*d.*, £66 14*s.* 7½*d.*, and £111 15*s.* 8*d.*

(327) £17 11*s.* 7½*d.*, £128 15*s.* 8¼*d.*, £372 4*s.* 3*d.*, £1234 9*s.* 4¾*d.*, and £2579 2*s.* 2½*d.*

(328) 17 cwt. 23 lbs. 7 oz. 14 drs., 1 qr. 8 lbs. 5 drs., 6 cwt. 3 qrs. 13 oz. 4 drs., 12 cwt. 3 qrs. 17 lbs. 8 oz. 15 drs., 10 lbs. 15 oz., and 19 cwt. 2 qrs. 6 lbs. 1 oz. 7 drs.

(329) 167 tons 4 cwt. 1 qr., 2 tons 7 cwt. 15 lbs. 9 oz., 18 cwt. 3 qrs. 12 lbs. 15 oz. 14 drs., 794 tons 2 qrs. 11 lbs. 10 oz. 9 drs., 27 lbs. 11 oz. 15 drs., and 85 tons 13 cwt. 3 qrs. 6 lbs. 8 oz. 2 drs.

(330) 715 tons, 8 tons 24 lbs., 3 qrs. 7 drs., 19 cwt. 19 lbs. 13 oz., 2 qrs. 11 oz. 15 drs., and 17 cwt. 8 lbs.

(331) 17 lbs. 7 oz. 17 dwts. 21 grs., 8 lbs. 11 oz. 6 dwts. 14 grs., 13 oz. 4 grs., 15 lbs. 9 oz. 12 dwts. 9 grs., 14 lbs., and 15 dwts. 23 grs.

(332) 9 oz. 12 dwts., 29 lbs. 18 dwts., 6 oz. 5 dwts. 22 grs., 17 dwts. 19 grs., and 12 lbs. 8 oz.

(333) 37 po. 3 yds. 1 ft. 11 in., 12 po. 3 yds. 10 in., 15 po. 3 in., 9 po. 4 yds. 7 in., 3 po. 1 in. and 2 po. 3 yds. 2 ft. 3 in.

(334) 14 m. 5 fur. 17 po. 2 yds. 1 ft. 6 in., 133 m. 33 po. 4 yds., 1 fur. 24 po. 5 yds. 6 in., 7 fur. 36 po., 3 m. 1 yd., and 43 m. 7 fur. 31 po. 1 yd. 1 ft. 6 in.

(335) 259 m. 7 fur. 37 po. 4 yds. 9 in., 3 m. 1 fur. 8 po. 1 ft. 3 in., 40 m. 6 fur. 20 po. 5 yds. 2 ft., 17 m. 20 po. 4 yds. 4 in., 175 m. 4 fur. 14 po. 5 yds. 1 ft. 8 in., and 3 m. 3 fur. 17 po. 2 yds.

(336) 37 p. 13 s. yds. 7 s. ft. 85 s. in., 22 p. 21 s. yds. 1 s. ft. 60 s. in., 5 p. 26 s. yds. 4 s. ft. 99 s. in., 17 p. 27 s. yds. 5 s. ft. 11 s. in., 9 p. 24 s. yds. 8 s. ft. 5 s. in., and 33 p. 19 s. yds. 3 s. ft. 36 s. in.

(337) 125 a. 3 r. 18 p. 25 s. yds., 2 r. 22 p. 19 s. yds., 13 a. 16 p. 28 s. yds., 407 a. 1 r. 7 p. 23 s. yds., 18 a. 27 s. yds., and 37 p. 24 s. yds.

(338) 37 r. 33 p. 23 s. yds. 7 s. ft. 121 s. in., 21 r. 25 p. 17 s. yds. 4 s. ft. 92 s. in., 56 r. 9 p. 28 s. yds. 8 s. ft. 101 s. in., 17 p. 30 s. yds. 6 s. ft. 9 s. in., 7 r. 15 s. yds. 3 s. ft. 72 s. in., and 38 p. 19 s. yds. 1 s. ft. 3 s. in.

(339) 17 qrs. 5 bus. 1 pk. 2 qts., 9 qrs. 7 bus. 3 pks. 1 gal. 3 qts., 125 qrs. 2 bus. 1 qt., 2 pks. 1 gal. 3 qts., 27 qrs. 6 bus. 3 pks., and 5 qrs. 3 bus. 1 pk. 1 gal. 2 qts.

(340) 12 lds. 4 qrs. 7 bus. 3 pks., 27 lbs. 3 qrs. 5 bus., 37 lds. 1 bus. 2 pks., 44 lds. 2 qrs. 6 bus. 1 pk., 7 lds. 2 qrs. 3 bus., and 1 qr. 2 pks.

Note.—An exercise in compound addition is given, with few exceptions, to every candidate for the English Civil Service. The following is a specimen of the papers used in examinations not competitive. Those used in competitive examinations are of the same character, but of greater length, containing twelve columns instead of six.

The time allowed is half an hour.

It is very important that the addition should be *quite correct*, and additional credit is also given for rapidity.

	£	s.	d.
(α)	786178	2	4
	810769	2	2
	152836	1	9
	538097	1	2
	82163	9	10
	86752	15	3
	74573	2	4
	90574	2	1
	531067	13	8
	98278	17	6
	99374	5	7
	113257	8	11
	23276	5	9
	381397	7	1
	423925	4	11
	213884	3	2

	£	s.	d.
(β)	197314	5	6
	34232	1	11
	361211	6	7
	593621	4	5
	19109	19	10
	90473	16	5
	318019	3	8
	297215	9	5
	627412	3	7
	418203	2	3
	945642	1	8
	536408	13	10
	86631	7	7
	60123	7	7
	83641	8	9
	961403	5	5

	£	s.	d.
(γ)	273144	9	9
	65211	6	8
	110732	17	1
	90486	3	7
	426409	12	8
	214321	18	9
	721403	7	3
	859276	3	8
	23997	5	2
	36197	11	7
	189603	2	5
	92485	9	7
	8542	18	9
	91376	2	11
	264103	7	7
	26179	5	8

(δ)	219432	2	6
	9632	2	6
	41708	7	7
	23109	19	10
	612356	1	2
	711791	6	1
	83479	2	8
	94712	3	3
	32752	15	3
	836729	6	7
	234526	1	1
	617173	8	6
	753276	5	9
	11835	0	3
	74321	18	9
	14203	5	6

(ε)	181437	9	8
	91467	1	3
	671646	3	11
	156917	11	9
	879	6	1
	159673	1	0
	42361	8	2
	16391	1	4
	471067	13	8
	692765	1	2
	754739	2	1
	927	6	8
	416228	10	9
	92176	5	5
	90704	8	8
	95316	2	2

(ζ)	211618	7	10
	127433	9	5
	25672	3	4
	239416	9	8
	181528	3	6
	109786	16	11
	274032	12	1
	514376	15	10
	83215	9	5
	14239	2	5
	352376	0	9
	319107	4	8
	2489	6	7
	13789	9	8
	96738	4	6
	310397	8	7

COMPOUND SUBTRACTION.

Find the difference between the following quantities:—

(341) **£1347 6*s.* 2¼*d.* and £928 17*s.* 2¾*d.***
(342) **£1052 13*s.* 7½*d.* and £761 18*s.* 10¾*d.***
(343) **£739 1*s.* 6*d.* and £650 9*s.* 5½*d.***
(344) **£543061 12*s.* 2½*d.* and £18921 15*s.* 3¾*d.***
(345) **£30608 14*s.* 2*d.* and £27876 16*s.* 7*d.***

(333) 37 po. 3 yds. 1 ft. 11 in., 23 po. 3 yds. 10 in., 15 po. 3 in., 9 po. 4 yds. 7 in., 3 po. 1 in. and 2 po. 3 yds. 2 ft. 3 in.

(334) 14 m. 5 fur. 17 po. 3 yds. 1 ft. 6 in., 130 m. 33 po. 4 yds 1 fur. 34 po. 5 yds. 6 in., 7 fur. 36 po., 3 m. 1 yd., an 43 m. 7 fur. 30 po. 1 yd. 1 ft. 6 in.

(335) 259 m. 7 fur. 37 po. 4 yds. 9 in., 3 m. 1 fur. 8 po. 1 ft. 3 in., 40 m. 6 fur. 30 po. 5 yds. 2 ft., 17 m. 39 po. 4 yds. 4 in., 175 m. 4 fur. 14 po. 5 yds. 1 ft. 8 in., and 3 m. 3 fur. 17 p 2 yds.

(336) 37 p. 13 s. yds. 7 s. ft. 85 s. in., 23 p. 21 s. yds. 1 s. ft. 60 s. in., 5 p. 26 s. yds. 4 s. ft. 99 s. in., 17 p. 27 s. yds. 5 s. ft. 11 s. in., 9 p. 24 s. yds. 8 s. ft. 5 s. in., and 33 p. 19 s. yds. 3 s. ft. 36 s. in

(337) 125 a. 3 r. 18 p. 25 s. yds., 2 r. 22 p. 19 s. yds., 13 a. 16 p. 28 s. yds, 437 a. 1 r. 7 p. 23 s. yds., 13 a. 27 s. yds., and 37 p. 24 s. yds

(338) 37 r. 33 p. 23 s. yds. 7 s. ft. 121 s. in., 21 r. 25 p. 17 s. yds. 4 s. f 92 s. in., 56 r. 9 p. 28 s. yds. 8 s. ft. 101 s. in., 17 p 30 s. yds. 6 s. ft. 9 s. in., 7 r. 15 s. yds. 3 s. ft. 72 s. in., an 38 p. 19 s. yds. 1 s. ft. 3 s. in.

(339) 17 qrs. 5 bus. 1 pk. 2 qts., 9 qrs. 7 bus. 3 pks. 1 gal. 3 qt, 125 qrs. 2 bus. 1 qt., 2 pks. 1 gal. 3 qts., 27 qrs. 6 bus. 3 pk, and 5 qrs. 3 bus. 1 pk. 1 gal. 2 qts.

(340) 12 lds. 4 qrs. 7 bus. 3 pks., 27 lds. 3 qrs. 5 bus., 37 lds. 1 bus. 2 pk 44 lds. 2 qrs. 6 bus. 1 pk., 7 lds. 2 qrs. 3 bus., and 1 qr. 2 pk

Note.—An exercise in compound addition is given, with few exce tions, to every candidate for the English Civil Service. The followi is a specimen of the papers used in examinations not competi Those used in competitive examinations are of the same characte, b of greater length, containing twelve columns instead of six.

The time allowed is half an hour.

It is very important that the addition should be quite correct, a additional credit is also g

	£	s.	d.		£	s.	d.		£	s.	d.
(α)	786178	2	4	(β)	197314	5	6	(γ)	273144	9	9
	810769	2	2		34232	1	11		65211	6	8
	152836	1	9		361211	6	7		110732	17	1
	538097	1	2		593621	4	5		90486	3	7
	82163	9	10		19109	19	10		426409	12	8
	86752	15	3		90473	16	5		214321	18	9
	74573	2	4		318019	3	8		721403	7	3
	90574	2	1		297215	9	5		859276	3	8
	531067	13	8		627412	3	7		23997	5	2
	98278	17	6		418203	2	3		36197	11	7
	99374	5	7		945642	1	8		189603	2	5
	113257	8	11		536408	13	10		92485	9	7
	23276	5	9		86631	7	7		8542	18	9
	381397	7	1		60123	7	7		91376	2	11
	423925	4	11		83641	8	9		264103	7	7
	213884	3	2		961403	5	5		26179	5	8

(δ)	219432	2	6	(ε)	181437	9	8	(ζ)	211618	7	10
	9632	2	6		91467	1	3		127433	9	5
	41708	7	7		671646	3	11		25672	3	4
	23109	19	10		156917	11	9		239416	9	8
	612356	1	2		879	6	1		181528	3	6
	711791	6	1		159673	1	0		109786	16	11
	83479	2	8		42361	8	2		274032	12	1
	94712	3	3		16391	1	4		514376	15	10
	32752	15	3		471067	13	8		83215	9	5
	836729	6	7		692765	1	2		14239	2	5
	234526	1	1		754739	2	1		352376	0	9
	617173	8	6		[illegible]	6	8		319107	4	8
	753276	5	9		[illegible]	[illegible]	9		2489	6	7
	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	5		13789	9	8
	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]		96738	4	6
	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]		310397	8	7

ACTION.

quantities :—

(346) 54 tons 8 cwt. 1 qr. 17 lbs. and 9 tons 16 cwt. 3 qrs. 22 lbs.

(347) 17 tons 8 cwt. 1 qr. 23 lbs. 7 oz. and 12 tons 9 cwt. 3 qrs. 22 lbs. 9 oz.

(348) 19 cwt. 2 qrs. 13 lbs. 10 oz. 14 drs. and 11 cwt. 2 qrs. 23 lbs. 12 oz. 15 drs.

(349) 57 lbs. 4 oz. 9 dwts. 18 grs. and 35 lbs. 9 oz. 16 dwts. 15 grs.

(350) 13 lbs. 7 oz. 15 dwts. 7 grs. and 11 lbs. 10 oz. 12 dwts. 20 grs.

(351) 169 lbs. 6 oz. 14 dwts. 17 grs. and 83 lbs. 7 oz. 15 dwts. 20 grs.

(352) 13 m. 6 fur. 35 po. 3 yds. 1 ft. 6 in. and 12 m. 38 po. 4 yds.

(353) 35 lea. 4 fur. 23 po. 4 yds. 1 ft. and 28 lea. 5 fur. 39 po. 5 yds. 6 in.

(354) 56 a. 2 r. 34 p. and 48 a. 3 r. 38 p.

(355) 3 r. 28 p. 27 s. yds. 7 s. ft. and 1 r. 39 p. 29 s. yds. 1 s. ft. 36 s. in.

(356) 23 lds. 2 qrs. 5 bus. 3 pks. and 18 lds. 2 qrs. 6 bus.

(357) A ship with its cargo is worth £21456 8*s.* 9*d.*, the value of the ship is £4978 19*s.* 10*d.*; find the value of the cargo.

(358) What sum added to £1367 17*s.* 4½*d.* will make £1500?

(359) The receipts from an exhibition are £2007 17*s.*; what will be the profits after allowing £1400 17*s.* 9*d.* for expenses?

(360) A person whose rental is £1157 9*s.* 8*d.*, has to pay interest on mortgages to the amount of £439 10*s.* a year; how much may he spend in the year so as to put by £220?

COMPOUND MULTIPLICATION.

Multiply

(361) £4238 6*s.* 4¾*d.* by 7; £24273 7*s.* 4¼*d.* by 9.

(262) £9479 7*s.* 5½*d.* by 8; £639 7*s.* 8¼*d.* by 28.

(363) £129 17*s.* 10¾*d.* by 108; £3509 1*s.* 2½*d.* by 39.

(364) £213 4*s.* 5½*d.* by 121; £4156 11*s.* 6½*d.* by 620.

(365) £279 13*s.* 3¾*d.* by 58; £317 9*s.* 6¼*d.* by 184.

(366) £173 15*s.* 1½*d.* by 157; £7053 11*s.* 6½*d.* by 52.

(367) £107 10*s.* 11½*d.* by 217; £7321 2*s.* 9¾*d.* by 97.

(368) £539 4*s.* 8¾*d.* by 416; £219 17*s.* 8¾*d.* by 725.

(369) £35 15*s.* 8¾*d.* by 26; £97 8*s.* 2¼*d.* by 67.

(370) £17 11*s.* 10¾*d.* by 135; £29 17*s.* 0½*d.* by 140.

(371) £19 13*s.* 7½*d.* by 137; £1763 7*s.* 11¼*d.* by 1119

(372) 3 tons 17 cwt. 27 lbs. 11 oz. by 76.

(373) 5 tons 3 cwt. 2 qrs. 4 lbs. by 39.

(374) 4 cwt. 3 qrs. 15 lbs. 7 oz. 11 drs. by 125.

(375) 27 lbs. 7 oz. 13 dwts. 23 grs. by 75.

(376) 7 fur. 23 po. 4 yds. 2 ft. 8 in. by 106.

(377) 37 m. 5 fur. 21 po. 3 yds. by 115.

(378) 108 a. 3 r. 34 p. by 93.

(379) What is the weight of 27 bars of gold, 10 of which weigh each 2 lbs. 3 oz. 17 dwts. and the others each 3 lbs. 20 grs.

(380) A man owes £137 17*s.* to each of 5 creditors, £23 7*s.* 10*d.* to each of 3 others, and £41 6*s.* 8*d.* to each of 4 others; at the same time 6 debtors owe him each £107 13*s.* 4*d.*, 2 others £84 10*s.* each, and 5 others £30 3*s.* 8*d.* each. How much will he have after settling his accounts?

COMPOUND DIVISION.

Divide, and prove by multiplication

(381) £2396 7*s.* 6*d.* by 6; £2682076 2*s.* 6*d.* by 12.

(382) £4367074 1*s.* by 72; £2 2*s.* 6*d.* by 17.

(383) £691 2*s.* 1$\frac{1}{2}$*d.* by 93; £3059678 17*s.* 1*d.* by 133.

(384) £214673 3*s.* 3$\frac{3}{4}$*d.* by 521; £3509768 16*s.* 1*d.* by 133.

(385) 21 tons 12 cwt. 1 qr. by 64; 5 yds. 2 ft. 11 in. by 12.

(386) By what number must 4 cwt. 2 qrs. 8 lbs. be multiplied so as to make 40 tons?

(387) What sum of money will be required per week of 7 days to pay 8 companies, each consisting of 95 men, at 1*s.* 1$\frac{1}{2}$*d.* a day per man?

(388) What length added to itself 58 times will make up 2665 m. 2 fur. 31 po. $\frac{1}{2}$ yd.?

(389) There are 2179 hackney carriages in London, which earn on an average 14*s.* a day each; what is the gross amount expended on these vehicles in a year?

(390) How much time, in the course of 33 years, will a person who rises at 5 o'clock in the morning gain over another who rises at 8 o'clock, supposing both to go to bed at 10 o'clock at night, and reckoning 365 days to the year?

Multiply

(391) £35 17*s.* 6$\frac{3}{4}$*d.* by 39$\frac{1}{5}$.

(392) £17 7*s.* 9$\frac{5}{8}$*d.* by $\frac{4}{7}$.

(393) £29 6*s.* 11$\frac{3}{4}$*d.* by 17$\frac{3}{5}$.

(394) £45 14*s.* 7$\frac{7}{12}$*d.* by 3$\frac{5}{6}$.

Divide

(395) £109 13*s.* 4$\frac{1}{4}$*d.* by 1$\frac{1}{4}$.

(396) £9075 11*s.* 2*d.* by 11$\frac{3}{7}$.

(397) £914 7*s.* 6$\frac{1}{2}$*d.* by 4$\frac{4}{5}$.

(398) £6484 9*s.* 10$\frac{2}{3}$*d.* by 8$\frac{1}{3}$.

(399) By what number must £28 15*s.* 9$\frac{7}{8}$*d.* be multiplied to make £32 18*s.* 1*d.*?

(400) By what number must £42 16*s.* 4$\frac{7}{8}$*d.* be divided to give £27 10*s.* 6$\frac{1}{2}$*d.* for the quotient?

REDUCTION OF FRACTIONS.

Find the value of

(401) $\frac{7}{8}$ of a shilling; $\frac{5}{8}$ of £1; $\frac{5}{14}$ of 5*s.* 10*d.*

(402) $3\frac{3}{8}$ of £5; $\frac{3}{20}$ of £4 10*s.*; $\frac{17}{19}$ of £6 13*s.*

(403) $3\frac{5}{14}$ of £1 11*s.* 6*d.*; $1\frac{7}{13}$ of 5*s.* $8\frac{1}{4}$*d.*; $\frac{5}{36}$ of a guinea.

(404) $\frac{35}{36}$ of £140 3*s.* 6*d.*; $\frac{5}{16}$ of 21*s.*; $2\frac{11}{12}$ of a moidore.

(405) £1 11*s.* 6*d.* $\div \dfrac{1\frac{2}{7}}{3\frac{1}{7}}$; $\dfrac{1\frac{8}{11}}{1\frac{2}{3}}$ of £8 8*s.* $5\frac{1}{4}$*d.*; $\dfrac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of 27*s.*

(406) $\frac{2}{3}$ of £1 + $\frac{3}{7}$ of £2 10*s.* 2*d.* + $2\frac{1}{3}$ of 16*s.*

(407) $\frac{5}{12}$ of £1 + $\frac{3}{7}$ of a guinea + $\frac{4}{13}$ of a shilling + $\frac{8}{13}$ of a halfpenny.

(408) $\frac{5}{7}$ of £1 + $\frac{3}{8}$ of a guinea + $\frac{3}{560}$ of 6*s.* 8*d.* + $\frac{1}{14}$ of half-a-crown.

(409) $\frac{13}{15}$ of a shilling + $\frac{11}{60}$ of half-a-crown + $\frac{2}{5}$ of £1 + $\frac{1}{6}$ of a guinea.

(410) $3\frac{5}{8}$ of £1 + $9\frac{3}{7}$ of a shilling + $2\frac{4}{5}$ of a penny.

(411) $\frac{5}{6}$ of 3 guineas + $\frac{3}{4}$ of 5*s.* 6*d.* + $\frac{1}{2}$ of $\frac{5}{6}$ of 5*d.*

(412) $1\frac{3}{4}$ of $3\frac{1}{2}$ of a moidore + $\frac{5}{8}$ of 13*s.* 4*d.* − $\frac{2}{7}$ of $1\frac{5}{9}$ of 6*s.* 8*d.*

(413) $\frac{1}{12}$ of £7 4*s.* − $\frac{17}{128}$ of £1 6*s.* 8*d.* + $\frac{17}{1375}$ of £2 17*s.* $3\frac{1}{2}$*d.*

(414) $\frac{2}{3}$ of $\frac{5}{8}$ of a crown + $\frac{3}{7}$ of $\frac{14}{15}$ of a shilling.

(415) $\frac{2}{7}$ of £1 + $\frac{47}{147}$ of a guinea + $\frac{211}{840}$ of 6*s.* 8*d.* + $\frac{5}{63}$ of a crown.

(416) $\frac{5}{7}$ of a ton; $\frac{2}{7}$ of a lb. troy; $1\frac{8}{17}$ of 2 tons 11 cwt. 1 qr. 6 lbs.

(417) $3\frac{4}{7}$ of 3 cwt. 3 qrs. 21 lbs. 1 oz. 5 drs.; $5\frac{3}{8}$ of 11 stone 6 lbs.

(418) $\frac{2}{5}$ of 27 po. 2 ft.; $10\frac{5}{12}$ of 3 A. 3 R. 3 P.

(419) $\frac{4}{7}$ of a cwt. + $8\frac{5}{6}$ lbs. + $3\frac{9}{10}$ oz.

(420) $7\frac{5}{13}$ of 1 ton 19 cwt. − $5\frac{7}{8}$ of 3 qrs. 8 drs.

(421) $\frac{7}{10}$ of a mile − $\frac{5}{8}$ of a fur. + $\frac{4}{11}$ of a pole.

(422) $\frac{7}{16}$ of an acre + $2\frac{1}{2}$ acres − $1\frac{7}{8}$ of an acre.

Reduce

(423) $7\frac{1}{2}$*d.* to the fraction of a crown; 2*s.* $6\frac{1}{2}$*d.* to the fraction of 1*s.* 6*d.*

(424) 9*s.* 10*d.* to the fraction of £1; 7*s.* $10\frac{1}{2}$*d.* to the fraction of £1.

(425) £1 7*s.* to the fraction of 30*s.*; £7 4*s.* 3*d.* to the fraction of £21 12*s.* 9*d.*

(426) £5 16*s.* 4*d.* to the fraction of £16 8*s.* 8*d.*; £7 17*s.* 6*d.* to the fraction of 9 guineas.

(427) £4 12*s.* $1\frac{1}{2}$*d.* to the fraction of £1 9*s.* $3\frac{3}{4}$*d.*; £$1\frac{1}{3}$ to the fraction of $1\frac{1}{3}$ guineas.

(428) 17*s.* 6*d.* to the fraction of £100; £7 9*s.* 6*d.* to the fraction of £13 14*s.* 6*d.*

(429) £8 9*s.* 3*d.* to the fraction of £21 12*s.* 9*d.*

(430) $2\frac{2}{3}$ half-guineas to the fraction of 10*s.* $11\frac{1}{4}$*d.*

(431) £11 6*s.* 5*d.* to the fraction of £10 5*s.* 4*d.*

(432) 1 of 13*s.* 4*d.* to the fraction of £5

(433) 3 qrs. 14 lbs. to the fraction of 1 ton 4 cwt. 14 lbs.
(434) 19 lbs. to the fraction of 2 qrs. 5 lbs. 8 oz.
(435) 5 yds. 1 ft. to the fraction of a mile.
(436) $5\frac{1}{5}$ s. yds. to the fraction of $1\frac{5}{8}$ P.
(437) 6 A. 1 R. 4 P. to the fraction of $2\frac{1}{2}$ R.
(438) 3 wks. 6 d. 16 hrs. to the fraction of 4 wks.
(439) What fraction is £1 of 12*s.* $4\frac{1}{2}$*d.*?
(440) What fraction of half-a-crown is $\frac{2}{9}$ of 6*s.* 8*d.*?
(441) What fraction is $\frac{3}{7}$ of £1 of $1\frac{1}{3}$ of £3 5*s.*?
(442) What fraction of 2 cwt. is 3 qrs. 4 lbs.?
(443) What fraction is $\frac{36}{37}$ of £17 2*s.* 3*d.* of £58 5*s.* 6*d.*?

Express

(444) $\frac{7}{11}$ of 3 qrs. 4 lbs. as the fraction of 1 cwt.
(445) $\frac{6}{13}$ of 8 oz. 3 dwts. 1 gr. as the fraction of 5 lbs. 5 oz.
(446) $7\frac{3}{5}$ of 3 A. 2 R. 5 P. as the fraction of $15\frac{2}{3}$ A.
(447) 1 s. yd. 2 s. ft. 36 s. in. as the fraction of 2 s. ft. 5 s. in.
(448) $3\frac{3}{7}$ of 4*s.* 1*d.* $-\frac{7}{8}$ of $1\frac{3}{5}$ of 4*s.* 2*d.* $+\frac{17}{25}$ of £6 5*s.* to the fraction of $\frac{5}{7}$ of 15 guineas.
(449) $\frac{3}{5}$ of £3 7*s.* 6*d.* $+\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ guineas to the fraction of £2.
(450) $\dfrac{3\frac{1}{9}}{1\frac{1}{13}}$ of $\{\frac{19}{120}$ of £1 $-\frac{7}{48}$ of 1*s.*$\}$ to the fraction of £1 6*s.*

REDUCTION OF DECIMALS.

Find the values of

(451) ·75 of £1; ·16875 of £1.
(452) ·375 of a guinea; 2·878125 of £1.
(453) ·03125 of £25; 7·345 of £1.
(454) 6·25 of 6*s.* 8*d.*; 7·125 of $3\frac{1}{2}$ guineas.
(455) 2·0875 of £1; ·0425 of £100.
(456) 2·625 of £1; ·432 of 13*s.* 4*d.*
(457) 12·875 of £3 5*s.* 6*d.*; 27·25 of 2*s.* 4*d.*
(458) 1·05625 of 6*s.* 8*d.*; 1·005 of 15 guineas.
(459) 8·9485 of £13 12*s.*; ·82525 of 10*s.* 2*d.*
(460) 3·00725 of £33 17*s.* 1*d.*; 7·575 of 13*s.* 8*d.*
(461) ·000895 of 3 moidores; 2·375 of a ton.
(462) ·03234 of a guinea; ·8473 of a cwt.
(463) 3·175 of 3 tons 8 cwt.; 3·625 of a cwt.
(464) 4·106 of 3 cwt. 1 qr. 21 lbs.; 5·128 of 7 cwt. 2 qrs.
(465) ·7 of 8 furlongs; 19·625 of 4 furlongs.
(466) 11·15 of 3 acres; 3·3125 of 8 wks. 4 d. 2 hrs.
(467) 2·35 of 3*s.*; ·0138 of £4 14*s.* 6*d.*

(468) $\cdot 38\dot{3}$ of £1; $1\cdot 5\dot{7}4\dot{0}$ of 27*s.*

(469) $\cdot 06\dot{3}$ of 500 guineas; $\cdot 16\dot{3}$ of $2\frac{1}{2}$ miles.

(470) $1\cdot 30\dot{4}$ of £1 17*s.* 6*d.*; $2\cdot 31\dot{6}$ of £1 10*s.*

(471) ·375 of a guinea + 2·1875 of a crown + ·3 of 7*s.* 6*d.*

(472) ·3211 of a guinea + $\frac{3}{8}$ of a crown + ·6925 of a shilling.

(273) ·68125 of £1 + ·375 of 13*s.* 4*d.* + ·605 of £3 2*s.* 6*d.*

(474) ·634375 of £1 + ·025 of 25*s.* + 2·05 of a shilling.

(475) 1·125 of a moi. + 44·045 of $11\frac{1}{4}$*d.* — ·0625 of 7*s.* 6*d.*

(476) ·175 of a ton + ·195 of a cwt. + ·145 of a quarter.

(477) ·573 of an inch + ·751 of a yard.

(478) 3·625 of £1 + $9\cdot\dot{4}2857\dot{1}$ of a guinea. — $\cdot\dot{2}$ of 6*s.* 9*d.*

(479) ·685 of 6*s.* 8*d.* — ·54635 of a moi. + ·796135 of £2 10*s.*

(480) 4·39845 of £5 + 1·875 of £1 10*s.* — 3·9825 of $7\frac{1}{2}$ crowns.

Reduce

(481) 18*s.* $8\frac{1}{4}$*d.* to the decimal of 2*s.* 2*d.*; 1*s.* 2*d.* to the decimal of 12*s.* $8\frac{1}{4}$*d.* to 4 places.

(482) £2 11*s.* $6\frac{3}{4}$*d.* to the decimal of £1 10*s.*; 9*s.* $4\frac{1}{2}$*d.* to the decimal of £1.

(483) 17*s.* 6*d.* to the decimal of £100; 15*s.* $5\frac{1}{2}$*d.* to the decimal of £1.

(484) 8*s.* $11\frac{1}{4}$*d.* to the decimal of a guinea; $\frac{3}{8}$ of a crown to the decimal of 21*s.*

(485) 7*s.* $3\frac{3}{4}$*d.* to the decimal of £1; 7*s.* $10\frac{1}{8}$*d.* to the decimal of £5.

(486) $\frac{3}{4}$ of half-a-crown to the decimal of £3; £27 10*s.* 1*d.* to the decimal of £40 5*s.*

(487) 3 cwt. 2 qrs. 14 lbs. to the decimal of a cwt.; $6\frac{2}{3}$ cwt. to the decimal of a ton.

(488) $\frac{1}{8}$ of 51 tons 9 cwt. 3 qrs. to the decimal of 5 cwt. 3 qrs.

(489) 2 tons 12 cwt. 3 qrs. 16 lbs. to the decimal of 2 cwt. 3 qrs. 4 lbs.

(490) 8 lbs. 10 oz. 4 dwts. to the decimal of 12 lbs.

(491) 2 oz. 3 dwts. 18 grs. to the decimal of 4 oz. 3 dwt. 8 grs.

(492) $5\frac{1}{2}$ inches to the decimal of $\frac{1}{4}$ of a mile; 2 R. 4 P. to the decimal of 1 R. 10 P.

(493) 6 furlongs to the decimal of a league; 3 R. 10 P. to the decimal of $2\frac{1}{2}$ acres.

(494) $5\frac{1}{2}$ pks. to the decimal of $3\frac{1}{2}$ quarters; $27\frac{1}{2}$ gallons to the decimal of $6\frac{2}{3}$ qts.

(495) $2\frac{1}{8}$ of $\frac{1}{10}$ of 40 yds. to the decimal of $\frac{1}{8}$ of 2 miles; 3 bus. 1 pk. to the decimal of a load.

(496) 2 yards of land, 21 A. 955 s. yds. 6 s. ft. 116 s. in. to the decimal of 4 yards of land 26 A. 752 s. yds. 3 s. ft. 36 s. in.

(497) Express the difference between $\frac{3}{5}$ of a guinea and $\frac{7}{9}$ of £1 as the

(498) Express $\frac{3}{8}$ of a crown + $\frac{4}{5}$ of a shilling as the decimal of 7*s.*

(499) Express $\frac{3}{16}$ of ·375 of ten shillings + $\frac{4}{5}$ of half-a-crown − $\frac{3}{4}$ of a shilling as the decimal of a pound.

(500) Find the value of $\cdot\dot{4}2857\dot{1}$ of £15 + $\frac{4}{5} \times \frac{1}{2\frac{4}{5}}$ of $\frac{1}{4}$ of £1 12*s.* + $\frac{3}{7}$ of 3*d.*, and express the result as the decimal of £20.

PRACTICE.

Find the value of

(501) 4084 articles at 2*s.* 6*d.* each; 3728 articles at 15*s.* each.

(502) 1035 articles at 6*s.* 8*d.* each; 9272 articles at 16*s.* 6*d.* each.

(503) 7804 articles at £1 7*s.* 6*d.* each; 5842 articles at £1 8*s.* each.

(504) 6554 articles at £7 13*s.* 4*d.* each; 2864 articles at £13 18*s.* 3*d.* each.

(505) 435865 articles at £1 13*s.* 8½*d.* each; 90748 articles at £19 7*s.* 2¼*d.* each.

(506) 217¼ yards of lace at £2 17*s.* 7½*d.* per yard.

(507) 28 cwt. 14 lbs. at 12*s.* 2*d.* per cwt.

(508) 30 cwt. 2 qrs. 14 lbs. at £1 17*s.* 5*d.* per cwt.

(509) 7 cwt. 3 qrs. 21 lbs. at £6 14*s.* 8*d.* per cwt.

(510) 3 tons 16 cwt. at £7 15*s.* 8*d.* per ton.

(511) 4 tons 5 cwt. 3 qrs. at £8 10*s.* per cwt.

(512) 15 tons 19 cwt. 3 qrs. 16 lbs. at £2 12*s.* 6*d.* per cwt.

(513) 7 cwt. 1 qr. 16 lbs. 12 oz. at £3 7*s.* 2*d.* per cwt.

(514) 3 qrs. 20 lbs. 10 oz. 4 drs. at 18*s.* 8*d.* per quarter.

(515) 5 cwt. 22 lbs. 14 oz. 8 drs. at £17 6*s.* per cwt.

(516) 72 cwt. 3 qrs. 17 lbs. at £7 4*s.* 6*d.* per cwt.

(517) 7 cwt. 1 qr. 15 lbs. 8 oz. at £17 0*s.* 9*d.* per cwt.

(518) 25 tons 3 cwt. 14 lbs. at £73 6*s.* 8*d.* per ton.

(519) 39 cwt. 1 qr. 20 lbs. at £6 2*s.* 6*d.* per cwt.

(520) 3 tons 18 cwt. 3 qrs. 12 lbs. at £2 17*s.* 9*d.* per cwt.

(521) 13 cwt. 3 qrs. 17 lbs. at £22 8*s.* per cwt.

(522) 19 cwt. 1 qr. 13 lbs. at £2 11*s.* 4*d.* per cwt.

(523) 9 tons 11 cwt. 2 qrs. 21 lbs. at £10 11*s.* 4*d.* per cwt.

(524) 14 cwt. 1 qr. 17 lbs. at 16*s.* 3*d.* per quarter.

(525) 56 tons 15 cwt. 2 qrs. 21 lbs. at £1 4*s.* 4*d.* per cwt.

(526) 15 oz. 6 dwts. 17 grs. at 5*s.* 10*d.* per ounce.

(527) 7 oz. 14 dwts. 21 grs. at £2 13*s.* 4*d.* per ounce.

(528) 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s.* 6*d.* per ounce.

(529) 5 lbs. 9 oz. 7 dwts. 12 grs. at £3 17*s.* 10*d.* per ounce.

(530) 11 lbs. 6 oz. 15 dwts. 11 grs. at £7 15*s.* 8*d.* per pound.

(531) 9 yds. 2 ft. 10 in. at 5*s.* 7½*d.* per yard.
(532) 11 miles 3 fur. 55 yds. of railway at £3500 per mile.
(533) 329 yds. 3 qrs. 2 nls. at £1 0*s.* 10*d.* per yard.
(534) 23 A. 3 R. 5 P. at £2 13*s.* 6*d.* per acre.
(535) 48 sq. yds 8 sq. ft. 114 s. in at £5 15*s.* per s. yard.
(536) 17 A. 3 R. 15 P. at £37 10*s.* per acre.
(537) 3 A. 1 R. 14 P. at £125 per acre.
(538) 35 A. 2 R. 30 P. at £30 1*s.* 8*d.* per acre.
(539) 25 A. 1 R. 10 P. at £42 2*s.* 4*d.* per acre.
(540) 2 R. 19 P. 12 s. yds. at £80 13*s.* 4*d.* per acre.
(541) 22 qrs. 6 bus. 3 pks. at £2 8*s.* per quarter.
(542) 9 qrs. 2 bus. 1 pk. at £2 16*s.* 8*d.* per quarter.
(543) 34 c. yds. 3 c. ft. 288 c. in. at £3 6*s.* 9*d.* per c. yard.
(544) What will be the dividend on £1740 5*s.* at 14*s.* 2*d.* in the pound?
(545) What will be the dividend on £1430 12*s.* at 13*s.* 4*d.* in the pound?
(546) Find the cost of 2 cwt. 1 qr. 10½ lbs. of soap at 3*s.* 9*d.* per stone.
(547) Find the amount of a man's wages for 3 wks. 4 d. 8 hrs. at 6*s.* 8*d.* a day, reckoning 10 hrs. to the day.
(548) Find the dividend on £7352 13*s.* 4*d.* at 12*s.* 6*d.* in the pound.
(549) What will be the rent of 67 acres of land for 2 years and 9 months, the yearly rent of the acre being £2 5*s.*?
(550) What will be the dividend on £2734 16*s.* 8*d.* at 9*s.* 4½*d.* in the pound?
(551) What will be the dividend on £2468 at 15*s.* 6*d.* in the pound?
(552) Find the cost of digging a trench the solid content of which is 5755 c. yds. at 15*s.* 7¾*d.* per cubic yard.
(553) How much will a creditor receive on a debt of £2485 at 17*s.* 9½*d.* in the pound?
(554) How much will the creditors lose by receiving 8*s.* 10¾*d.* in the pound from a bankrupt's estate who owes £4970?
(555) What must a person whose property is rated at £430 14*s.* contribute to a poor rate of 10*d.* in the pound?
(556) How much will 13 cwt. 3 qrs. 11 lbs. of sugar cost at £2 6*s.* 8*d.* per cwt.?
(557) What will be the dividend on a debt of £2574 15*s.* at 13*s.* 5*d.* in the pound?
(558) Find the value of 105 pockets of hops each weighing 1 cwt. 1 qr. 8 lbs.. at £4 12*s.* 6*d.* per cwt.

) Find the value of 15 silver plates each weighing 7 oz. 11 dwts. 6 grs., at 6*s.* 8*d.* per ounce.

) Find the value of 70 bales of cotton, each weighing 1 cwt. 1 qr. 21 lbs., at £2 3*s.* 1½*d.* per cwt.

) An officer's pay is 12*s.* 3*d.* a day; how much is that in a year?

) A bankrupt owes £25962 10*s.* What must his assets be worth in order that he may pay 7*s.* 11½*d.* in the pound?

) Find the land tax on a rental of £135 17*s.* 6*d.* at 1*s.* 3*d.* in the pound.

) How much income tax will a person whose income is £775 13*s.* pay in three years at 14*d.* in the pound?

SINGLE RULE OF THREE.

) If 17 yards of cloth are worth £6 3*s.* 3*d.*, what is the value of 68 yards of the same?

) If 1 bushel costs 5*s.* 10*d.*, how much can I buy for £27 5*s.* 5*d.*?

) If a ton of potatoes costs £7, what will be the price of 24 lbs.?

) A man walks 17 miles 1650 yards in 3 hours 45 minutes; what is his rate per hour?

) How many yards of linen at 4*s.* 1*d.* per yard must be given in exchange for 125 yards of cloth at 17*s.* 4¼*d.* per yard?

) If 7 cwt. 3 qrs. 17 lbs. 8 oz. cost £2 5*s.*, how much will 1 ton 11 cwt. 2 qrs. 14 lbs. cost?

) If 5 oz. 7 dwts. 12 grs. cost £5 7*s.* 6*d.*, how much will 1 lb. cost?

) A train which travels at the rate of 25 miles an hour is 4 hrs. 30 min. going from London to Birmingham; at what time will a train which leaves London at 2 P.M., and travels at the rate of 30 miles an hour, reach Birmingham?

) If 5 gallons of oil cost 18*s.* 4*d.*, find the cost of 13 gals. 3 qts. 1 pt.

5) If the carriage of 16 cwt. 3 qrs. 7 lbs. for 105 miles cost £1 15*s.*, how much will the carriage of 2 cwt. 1 qr. 17 lbs. cost for the same distance?

6) The cost of making a railway 27¾ miles long was £97125; what was the cost per mile?

7) A bankrupt's debts amount to £1864, and his assets to £699, how much will his creditors receive in the pound?

8) A man working 6 hours a day can finish a piece of work in 7½ days; how many hours a day must he work to finish it in 5 days?

(579) If 5 shillings worth of copper weighs 4 lbs., what will be the weight of 9 guineas worth?

(580) If a man walks $4\frac{1}{2}$ miles in an hour and twenty minutes, how long will he be walking $17\frac{3}{4}$ miles?

(581) A bankrupt's debts amount to £4586 8*s.*, and his effects to 3822 guineas; how much will his creditors receive in the pound?

(582) A bankrupt pays a dividend of 6*s.* 8*d.* in the pound; what is the loss of a creditor to whom he owes £750?

(583) If 48lbs. of potatoes can be bought for 3*s.*, how much will a ton of potatoes cost?

(584) If a horse trots $23\frac{3}{4}$ miles in $2\frac{1}{2}$ hours, what is his rate per hour?

(585) If $4\frac{1}{8}$ cwt. of sugar costs 21 guineas, what must be paid for $195\frac{1}{4}$ lbs.?

(586) How many acres will 34 men reap in the time that 10 men are reaping 23 acres?

(587) A fortress is provisioned for 3 weeks, at the rate of 15 oz. a day to each man; if only $10\frac{1}{2}$ oz. are served out daily to each man, how long can the place hold out?

(588) If a labourer's wages for $17\frac{3}{4}$ days amounts to £2 4*s.* $4\frac{1}{2}$*d.*, what will be the amount of his wages for $68\frac{3}{4}$ days?

(589) How many yards of cloth can I buy for £20 6*s.* 8*d.*, if 3 English ells cost £3 3*s.* 4*d.*?

(590) A bankrupt's assets amount to £225, out of which he pays 5*s.* in the pound on half his debts, and 4*s.* in the pound on the other half; find the amount of his debts.

(591) A broker sells £8640 stock; how much will his commission amount to at 2*s.* 6*d.* per cent.?

(592) If $12\frac{9}{11}$ oz. troy cost £12 8*s.*, how much will $10\frac{15}{22}$ lbs. cost?

(593) If a man walks $6\frac{1}{2}$ miles in $1\frac{1}{2}$ hours, in what time will he walk $16\frac{1}{2}$ miles?

(594) If $2\frac{1}{2}$ tons can be carried 76 miles for 19*s.*, how far ought 19 cwt. to be carried for the same?

(595) If I lend a friend £234 13*s.* 4*d.* for a year and a half, for how long ought he to lend me £176 to return the favour?

(596) A railway train travels $\frac{1}{4}$ of a mile in 18 seconds; how many miles an hour does it travel at this rate?

(597) What will be the income tax on £945 17*s.* 6*d.* at 14*d.* in the pound?

(598) A bankrupt's effects amount to 2548 guineas and his debts to £3057 12*s.*; how much will his creditors receive in the pound?

(599) What will be the income tax on £1405 15*s.* $11\frac{1}{4}$*d.*, at 1*s.* 4*d.* in the pound?

(600) Convert 44 lbs. 11 oz. 11 dwts. 16 grs. troy into avoirdupois weight. (1 lb. troy : 1 lb. avoirdupois :: 144 : 175.)

(601) If $3\frac{1}{2}$ lbs. cost 2*s.*, how much will 54 cwt. 2 qrs. 7 lbs. of sugar cost?

(602) A hawker buys 275 yards of cloth for £91 13*s.* 4*d.*; for how much must he sell 7 English ells so as to gain at the same rate £20 6*s.* 8*d.* on the whole?

(603) What is the income corresponding to an income tax of £108 1*s.* $4\frac{1}{2}$*d.*, at the rate of 9*d.* in the pound?

(604) If the price of 3 bushels of wheat is 16*s.* 9*d.*, find the price of 12 qrs. 2 bus. 1 pk.

(605) If 16 Napoleons are worth £12 14*s.* 8*d.*, what is the value of 105 Napoleons?

(606) A person who values his property at £3500 insures half of it at 5*s.* 6*d.* per cent. and half at 4*s.* 6*d.* per cent.; how much does it cost him?

(607) Find the income corresponding to an income tax of £50 7*s.* 1*d.*, at 5*d.* in the pound.

(608) A rate of 1*s.* 5*d.* in the pound is levied in a county whose rateable rental is £360817 10*s.*; find the amount produced.

(609) If a pocket of hops, weighing 1 cwt. 3 qrs. 12 lbs., cost £7 3*s.*, what is the price per cwt.?

(610) The shares of a given railway are at $59\frac{1}{2}$ when consols are at $93\frac{1}{2}$; what should be their value when consols are at $71\frac{1}{2}$?

(611) A coach travels 9 miles an hour, and a railway train goes 23 miles while the coach goes 11; how much time will be saved on a journey of 138 miles by taking the train instead of the coach?

(612) If $\frac{13}{17}$ cwt. cost £14 6*s.*, what must be paid for $\frac{7}{11}$ of a ton?

(613) If £59 10*s.* will buy 25 cwt. 2 qrs. of sugar, how much will £6 14*s.* 2*d.* buy?

(614) If the penny loaf weighs 5 oz. when wheat is at 62*s.* the quarter, what ought it to weigh when wheat is at 64*s.*?

(615) If by paying down £70 17*s.* 6*d.* a person becomes entitled to £2 9*s.* 7*d.* a year, what income would he have by paying down £2151 17*s.* 8*d.* in the same way?

(616) If the tax on an income of £1132 amount to £136 15*s.* 8*d.*, what is it in the pound?

(617) If the carriage of 107 cwt. 10 lbs. cost £37 9*s.* $7\frac{1}{2}$*d.*, what would the carriage of a ton cost?

(618) What is the income of a person who pays an income tax of £77 1*s.* $3\frac{3}{4}$*d.* when the rate is 10*d.* in the pound?

(619) How many yards of cloth, worth 3*s.* $7\frac{1}{2}$*d.* a yard, must be given in exchange for $935\frac{1}{2}$ yards, worth 18*s.* $1\frac{1}{2}$*d.* per yard?

(620) If a rental of £8050 per annum be charged with a land tax at 11*s.* 5*d.* per £100, what will be the net income?

(621) If when wheat is at 11*s.* 3*d.* the bushel, the sixpenny loaf weighs 28 oz., how much should it weigh when wheat is at 84*s.* the quarter?

(622) If $\frac{13}{15}$ of a cwt. be worth £7 3*s*, what will be the value of $\frac{6}{11}$ of a ton?

(623) When the income tax was 7*d.* in the pound a person paid £24 7*s.* 8*d.*; what additional sum does he pay when the income tax is 16*d.* in the pound?

(624) Gold of the value of £423267 arrives from Australia; what is its weight, the price being £3 18*s.* per ounce?

(625) A grocer buys half a ton of sugar for £30, and sells it at $7\frac{1}{2}$*d.* per lb., how much does he gain?

(626) If $3\frac{3}{4}$ lbs. of tea cost $\frac{11}{16}$ of £1, how many pounds may be bought for £$11\frac{11}{12}$?

(627) If $\frac{2}{3}$ of a ship be worth £220, what is the value of $\frac{3}{11}$ of the same?

(628) If a cask of wine containing 84 gallons cost £112 5*s.*, how much will be gained by selling it at £1 11*s.* 6*d.* per gallon?

(629) If $12\frac{1}{4}$ yards of cloth cost £7 3*s.* $7\frac{1}{4}$*d.*, how much will 107 English ells cost?

(630) If $8\frac{1}{4}$ cwt. cost 42 guineas, what will be the price of $97\frac{5}{8}$ lbs.?

(631) If 27 cwt. 1 qr. 3 lbs. 8 oz. of sugar cost £87 6*s.*, what will be the price of 7 lbs?

(632) If the yearly profits of an investment be £11 9*s.* 6*d.* per cent, how much must be invested to produce an income of £640 13*s.* 9*d.* a year?

(633) If a person spends £7 16*s.* in 13 days, how much will he save at that rate in a year out of an income of 219 guineas?

(634) A coach goes from London to Liverpool, at the rate of 9 miles an hour, in 24 hours; in what time shall I get from London to Liverpool by a train going 32 miles an hour?

(635) A besieged town which contains 22400 inhabitants has provisions for 3 weeks; how many must be sent away that it may hold out 7 weeks?

DOUBLE RULE OF THREE.

(636) If the keep of 14 horses for 24 days cost £28, how many horses may be kept 14 days for £35?

(637) If 16 men earn £15 12*s.* in 18 days, how many men will earn £35 2*s.* in 24 days?

(638) If 10 horses consume 7 bus. 2 pks. of corn in 7 days, in what time will 28 horses consume 3 qrs. 6 bus.?

(639) If the carriage of 8 cwt. of goods for 124 miles cost 6 guineas, what weight ought to be carried 31 miles for half the money?

(640) If 13 men earn £7 6*s.* 3*d.* in 7½ days, how much will 10 men earn in 11¾ days?

(641) If 7 men can mow 84 acres in 12 days of 8¼ hours each, how many acres can 30 men mow in 11 days of 7 hours 48 minutes each?

(642) If when wheat is at 60*s.* a quarter the sixpenny loaf weighs 4 lbs., how much should be paid for 25 lbs. of bread when wheat is at 40*s.* a quarter?

(643) If £3 7*s.* 6*d.* be paid for the carriage of 15¾ cwt. for 108 miles, how far ought 48 cwt. to be carried for 5 guineas?

(644) If the sixpenny loaf weighs 3½ lbs. when flour is 35*s.* a sack, what is the price of flour per sack when an eightpenny loaf weighs 4 lbs.?

(645) If 5 men receive £18 15*s.* for 12 weeks wages, what will the wages of 16 men for 20 weeks amount to?

(646) If 5 men can do a piece of work in 27 days, how long will 6 men be doing double the quantity?

(647) If the carriage of 65 lbs. for 120 miles cost 6*s.*, how much can be sent 750 miles for a guinea?

(648) If 780 men on full rations consume 91 quarters of wheat in 49 days, how many on half rations will consume 69 quarters in 9 days?

(649) If 21 men take 8 days to dig 20 acres, how many acres will 16 men dig in 12 days?

(650) If 6 iron bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lbs., how much will 15 bars weigh, each 6½ ft. long, 4 in. broad, and 3 in. thick?

(651) If 90*s.* will pay 5 men for 12 days work, how much will be required to pay 32 men for 24 days work?

(652) If 24 men working 8 hours a day can build a house in 70 days, in how many days could it be built by 42 men working 10 hours a day?

(653) If 40 panes of glass, each 2 ft. long and $1\frac{1}{2}$ ft. wide; cost £3 8*s*., what will be the cost of 34 panes measuring each $2\frac{1}{2}$ ft. long by 1 ft. wide?

(654) If 49 men can do a piece of work in 130 days of 8 hours each, how many hours a day must 196 men work to do one-fifth as much again in 26 days?

(655) If 20 horses and 96 sheep can be kept 18 days for £151 10*s*., what sum must be paid for 15 horses and 76 sheep for 8 days, supposing 5 horses to eat as much as 76 sheep?

(656) If 1600 soldiers consume 10 sacks of flour in 12 days, how many soldiers will consume 30 sacks in 4 days?

(657) If 288 men can dig a trench 230 yards long, 3 ft. wide, and 2 ft. deep in 5 days of 11 hours each, in how many days of 9 hours each will 22 men dig a trench 460 yards long, 5 ft. wide, and 3 ft. deep?

(658) If a party of 5 persons spend £641 13*s*. 4*d*. in 11 months, how much at the same rate would it cost a party of 7 persons for 4 months?

(659) If the corn of 13 horses for 63 days cost £17 6*s*. 8*d*. when corn is 4*s*. per bushel, how many horses will cost £10 13*s*. 4*d*. for corn in 56 days when corn is 4*s*. 6*d*. per bushel?

(660) If the wages of 25 men amount to £76 13*s*. 4*d*. in 16 days, how many men must work 24 days to receive £103 10*s*., the daily wages of the latter being one half those of the former?

(661) If 15 men can mow 300 acres of grass in 17 days, how long will 12 men be mowing 240 acres?

(662) If 6 horses can plough $17\frac{1}{2}$ acres in 4 days, how many acres will 54 horses plough in $2\frac{1}{4}$ days?

(663) If 12 men working 10 hours a day can do a piece of work in 9 days, in how many days will 15 men working 8 hours a day do a piece of work 3 times as great?

(664) If 24 men can saw 90 loads of wood in 6 days, working 9 hours a day, how many loads can 8 men saw in 36 days, working 12 hours a day?

(665) If 616 men can excavate a railway cutting $1\frac{1}{2}$ miles long and 33 ft. wide, its average depth being 21 ft., in 5 weeks, working 11 hours a day, in how long can 70 men dig a canal 3 miles long, 5 ft. deep, and 7 yards wide, working 10 hours a day?

(666) If 5 men can build a wall 9 ft. high and 12 yards long in 24 days, working 9 hours a day, how many men can build a wall 7 ft. high and 40 yards long in 10 days, working 8 hours a day?

(667) If 988 men on full rations consume 221 quarters of wheat in 119 days, how many on half rations will consume 357 quarters in 57 days?

(668) If the carriage of 37 st. 6 lbs. for 7 miles costs £2 5*s*., what weight ought to be carried 12 miles for £3 10*s*.?

(669) If 23 men can do a piece of work in 27 days, working 8 hours a day, how many hours a day must 30 men work to do one-third of the same in 5 days?

(670) If 17 men can build a wall 22 yards long, 5 ft. high, and 1 ft. wide in 9 days, working 11 hours a day, in how many days can 51 men build a wall 60 yards long, 7 ft. high, and 2 ft. wide, working 12 hours a day?

(671) Four men working 8 hours a day take 22 days to pave a road 440 yards long and 35 ft. wide; how many days will 4 men, 2 of whom work 8 hours a day and 2, 10 hours a day, take to pave a road 1575 yards long and $36\frac{1}{2}$ ft. wide?

(672) If 36 men, working 8 hours a day, can dig a trench 72 yards long, 18 wide, and 12 deep in 16 days, in how many days will 32 men, working 12 hours a day, dig one 64 yards long, 27 wide, and 18 deep?

(673) If a garrison of 600 men have provisions for 5 weeks, allowing each man 12 oz. per day, how many men can be maintained 10 weeks by the same quantity if each man is limited to 8 oz. per day?

(674) If 17 men can reap 24 acres in 27 days, how many acres will 18 men reap in 33 days?

(675) What will be the interest of £500 for 4 years if the interest of £600 for 1 year is £42?

(676) A canal 10 miles long is 8 yards wide at the top, 6 yards wide at the bottom, and 5 feet deep; how long would 800 men take to excavate it, each removing on an average 15 c. yards a day?

(677) If 24000 yards of cotton cloth, $\frac{3}{4}$ yd. wide, be worth £400 when the raw cotton is at $4\frac{1}{2}$*d.* a pound, what is the value of 36000 yds. of cotton cloth $1\frac{1}{4}$ yds. wide when the raw cotton is at 9*d.* a pound?

(678) If 48 pioneers, in 5 days of $12\frac{1}{2}$ hours each, can dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep, how many hours a day must 180 pioneers work during 84 days in order to dig a trench $4910\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{8}$ yards deep?

(679) If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench $232\frac{1}{2}$ yards

long, 3⅔ wide, and 2⅓ deep, in how many days of 9 hours each will 24 men dig a trench 337½ yards long, 5⅗ wide, and 3½ deep; supposing that the same number of men in the same time would be $\frac{7}{4}$ths as long digging the first as they would be in digging a trench of equal dimensions of the nature of the second?

(680) If 7 men can do a piece of work in 12 days of 10 hours each, how many men will do a piece of work 4 times as large in a fifth part of the time, if they work the same number of hours in the day, supposing that 3 of the second set can do as much work in an hour as 5 of the first set?

AREAS. SOLID CONTENTS.

(681) The sides of a rectangular plot are 57 ft. 7 in. and 43 ft. 5 in.; find its area.

(682) How many yards of matting, 1 yard wide, will be required for a room 36 ft. long and 8 yds. 2 ft. wide?

(683) What will be the expense of paving a yard 42 ft. 4 in. long and 12 ft. 9 in. wide, at 4*s.* 6*d.* per s. yard?

(684) How much will it cost to cover a room 16½ yards long and 11¼ yards wide with matting 1⅛ yd. wide, at 1*s.* 4*d.* a yard?

(685) How much paper, ⅞ of a yard wide, will be required to paper a room 27 ft. 5 in. long, 14 ft. 7 in. wide, and 12 ft. 10 in. high?

(686) How many yards of paper, 29 in. wide, will be required to paper a room 22 ft. 9 in. long, 17 ft. 3 in. wide, and 9 ft. 8 in. high?

(687) Find the cost of papering a room 16 ft. long, 11 ft. wide, and 10 ft. high, with paper 30 in. wide, at 7½*d.* a yard.

(688) How many yards of paper, 2½ ft. wide, will be required to paper a room 29 ft. 5 in. long, 20 ft. 7 in. wide, and 7½ ft. high?

(689) The breadth of a rectangular field is 26 yds. 2 ft. 8 in. and the area is an acre; find its length.

(690) How many yards of paper, ⅞ yd. wide, will be required to paper a room whose length is 26 ft. 4 in., breadth 18 ft. 8 in., and height 12 ft. 3 in.?

(691) How many yards of carpet, ½ a yard wide, will be required for a room 36 ft. long and 8 yds. 2 ft. wide?

(692) How many planks, 12 ft. long and 6 in. wide, will be required to floor a room 20 yards long and 15 yards wide?

(693) How many stones, each 1 yd. 1 ft. wide and 2 yds. long, will be required for a pavement 4 yds. wide down two sides of a

(694) Find the content of a square cistern whose side is $6\frac{3}{4}$ ft. and depth 8 ft.

(695) How many yards of matting, $3\frac{4}{5}$ ft. wide, will be required to cover a room 28 ft. 6 in. long and 20 ft. 8 in. wide?

(696) How many yards of paper, $1\frac{2}{3}$ yds. wide, will be required to paper a room 13 yds. 1 ft. long, 10 yds. wide, and $8\frac{1}{2}$ ft. high, allowing for 6 windows each 5 ft. by 3 ft., and a door 7 ft. by 5 ft.?

(697) What will be the cost of papering a room $16\frac{1}{2}$ yds. long, $11\frac{1}{4}$ yds. wide, and 10 ft. high, allowing for 3 windows, each 6 ft. by 5 ft., and 2 doors, each 9 ft. by 4 ft., with paper $1\frac{5}{6}$ yds. wide, at $5\frac{1}{2}$*d.* a yard?

(698) The sides of a rectangular field are 253 yards and a quarter of a mile; how many acres does it contain?

(699) How much paper, $\frac{3}{4}$ of a yard wide, will be required for a room 22 ft. long, 14 ft. wide, and 9 ft. high, allowing for 3 windows and 2 doors, each 6 ft. by 3 ft.?

(700) The edge of a cube is 17 inches; find its content in cubic feet and inches.

(701) If a c. foot of timber weighs 21 lbs., how much will a square stick 6 yds. 2 ft. long and 2 ft. wide weigh?

(702) How many copies of a book, 7 in. long, $5\frac{1}{2}$ in. wide, and 3 in. thick, can be packed in a box $3\frac{1}{2}$ ft. long, $2\frac{3}{4}$ ft. wide, and 1 ft. deep?

(703) A cubic foot of water weighs 1000 oz.; what will be the weight of water in a cistern 4 yds. 2 ft. long, 4 yds. wide, and 3 yds. 1 ft. deep?

(704) If 56 c. ft. 1404 c. in. of timber are required to floor a room 29 ft. 3 in. by 25 ft. 3 in., what is the thickness of the boards?

(705) A stone 18 in. long, 17 wide, and 7 deep, weighs 228 lbs., how many c. ft. of this kind of stone will freight a vessel of 230 tons burden?

(706) What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. wide, and 8 ft. 3 in. deep, when one $1\frac{2}{3}$ yds. long, $1\frac{1}{4}$ wide, and 2 ft. 6 in. deep, weighs 7500 lbs.?

(707) How many yards of paper, $\frac{2}{3}$ yd. wide, will be required to paper a room 30 ft. 9 in. long, 23 ft. 3 in. wide, and 9 ft. 6 in. high, allowing for four windows, each 4 ft. 3 in. by 3 ft., and a door 7 ft. 4 in. by 4. ft. 6 in.?

(708) A garden roller is 6 ft. 6 in. in circumference and 2 ft. 3 in. wide, and makes 12 revolutions as it rolls from one end of a lawn to

the other; after rolling 5 times up and down it has passed over the whole lawn once. Find the area of the lawn.

(709) How much wood will be required to make a box 3 ft. 4·54 in. long, 1 ft. 8·825 in. wide, and 2 ft. 6·62 in. deep?

(710) How many gallons will a cistern hold which is 8 ft. long, 5 ft. 6 in. wide, and 4 ft. 9 in. deep? Vide Art. **79.**

(711) How many feet must be cut off a circular and uniform timber stick, whose diameter is $1\frac{3}{4}$ ft., to get 11 c. ft. of timber? {The area of a circle $= 3\frac{1}{7} \times (\text{radius})^2$}.

(712) How many bricks, each ·75 ft. by ·375 ft., will be required to pave a yard 15·208$\dot{3}$ ft. long and 10·5 ft. wide?

(713) If a square foot of bark weighs $3\frac{1}{2}$ lbs., what weight of bark will there be on a timber stick 6 yds. 1 ft. long, and having an average diameter of 3 ft.? (The circumference of a circle $= 3\frac{1}{7} \times$ the diameter.)

(714) What weight of water will be contained in a cistern 5 ft. 4 in. deep, $5\frac{3}{8}$ ft. wide, and 4 yds. long, supposing a cubic foot of water to weigh 1000 oz.?

(715) What will be the cost of painting a room 20 ft. 3 in. long, 18 ft. 6 in. wide, and 10 ft. high, containing 2 windows, each 7 ft. by 4 ft., at 2*s.* 6*d.* per s. yard?

(716) A river, 30 ft. deep and 200 yards wide, discharges itself into the sea at the rate of 4 miles an hour; how many tons of water are discharged in 7 minutes? (714.)

(717) What will be the cost of papering a room 12 ft. 4 in. high, 16 ft. long, and 14 ft. 3 in. wide, with paper 2 ft. 6 in. wide, at 2*s.* 6*d.* a yard; the workmen also charging $\frac{1}{4}$*d.* for every square foot?

(718) The content of a cistern is the sum of two cubes whose edges are 10 in. and 2 in., and the area of its base is the difference of two squares whose sides are $1\frac{1}{3}$ ft. and $1\frac{2}{3}$ ft.; find its depth.

(719) Find the cost of a column 2 ft. in diameter and 4 yds. 2 ft. high, at 15*s.* 6*d.* per cubic foot. (711.)

(720) Find the superficies of a cylindrical column 49 ft. high and $4\frac{1}{4}$ ft. in diameter. (713.)

DUODECIMALS.

Multiply by the method of duodecimals, and express the results in cubic feet and inches.

(721) 12 ft. 4 in. by 3 ft. 5 in.; 15 ft. 7 in. by 5 ft. 11 in.

(722) 18 ft. 9 in. by 14 ft. 7 in.; 23 ft. 8 in. by 16 ft. 9 in.

(723) 27 ft. 6 in. 9 pts. by 5 ft. 3 in.; 4 ft. 6 in. 5 pts. by 9 ft. 4 in. 7 pts.
(724) 3 ft. 1 in. 11 pts. by 2 ft. 6 in. 7 pts. and the product by 1 ft. 7 in.
(725) 3 ft. 1 in. 11 pts. by 4 ft. 6 in. 8 pts. and the product by 2 ft. 7 in.
(726) 7 ft. 5 in. 8 pts. by 9 ft. 4 in. 11 pts.
(727) 2 ft. 7 in. 11 pts. by 3 ft. 5 in. 7 pts.
(728) 46 ft. 5 in. 4 pts. by 9 ft. 7 in. 8 pts.
(729) 6 ft. 7 in. 5 pts. by 8 ft. 3 in. 10 pts.
(730) 5 ft. 3 in. by 6 ft. 7 in. and the product by 7 ft. 11 in. What is the actual value of the denomination which occupies the place of inches in this product?

INTEREST AND DISCOUNT

Find the simple interest of

(731) £1000 for 2 years, at $4\frac{1}{4}$ per cent.
(732) £375 for 3 years and 8 months, at $3\frac{1}{2}$ per cent.
(733) £2175 10*s.* for 3 years, at $4\frac{1}{4}$ per cent.
(734) £4164 10*s.* for 6 years, at $2\frac{3}{4}$ per cent.
(735) £5856 11*s.* 5*d.* for 4 years, at $3\frac{1}{2}$ per cent.
(736) £98 15*s.* 10*d.* for 6 months, at $2\frac{1}{2}$ per cent.
(737) £999 19*s.* $9\frac{3}{4}$*d.* for 25 years, at 2 per cent.
(738) £7864 12*s.* 6*d.* for 5 years, at $3\frac{1}{2}$ per cent.
(739) £2427 8*s.* 8*d.* for 4 years 6 months, at $5\frac{1}{2}$ per cent.
(740) £1824 18*s.* 9*d.* for 10 months, at $6\frac{1}{2}$ per cent.
(741) £1923 15*s.* for 2 years 8 months, at $3\frac{3}{8}$ per cent.
(742) £1049 16*s.* 6*d.* for 6 years 4 months, at $4\frac{1}{2}$ per cent.

Find the amount, at simple interest, of

(743) £139 12*s.* 6*d.* for $3\frac{1}{2}$ years, at $5\frac{1}{2}$ per cent.
(744) £14797 10*s.* 8*d.* for 6 years 8 months, at 4 per cent.
(745) £784 6*s.* 8*d.* for 3 years 4 months, at 3 per cent.
(746) £895 13*s.* 4*d.* for $2\frac{1}{2}$ years, at 4 per cent.
(747) £1005 for 6 years, at 4 per cent.
(748) £1144 for 5 years 5 months, at $4\frac{1}{2}$ per cent.

Find the compound interest of

(749) £720 for 3 years, at 5 per cent.
(750) £1562 10*s.* for 3 years, at 4 per cent.
(751) £1220 14*s.* $0\frac{3}{4}$*d.* for $4\frac{1}{2}$ years, at 4 per cent.
(752) £125 5*s.* for 3 years, at 4 per cent.
(753) £275 for 3 years, at 5 per cent.

Find the amount, at compound interest, of

(754) £933 6*s.* 8*d.* for 3 years, at $2\frac{1}{2}$ per cent.
(755) £7755 for 4 years, at 5 per cent.

(756) £750 for 3 years, at $5\frac{1}{2}$ per cent.

(757) £230 15*s.* for 3 years, at 5 per cent.

(758) £7000 for 4 years, at 5 per cent.

(759) £5000 for 4 years, at 3 per cent.

(760) £4800 for 3 years, at $\frac{3}{8}$ per cent.

(761) At what rate per cent., simple interest, will £157 15*s.* 4*d.* amount to £295 16*s.* 3*d.* in 25 years?

(762) What sum of money will amount to £635 6*s.* 3*d.* in 6 years, at $5\frac{5}{8}$ per cent.?

(763) What sum of money will amount to £2264 12*s.* $2\frac{1}{4}$*d.* in 7 years, at $3\frac{1}{4}$ per cent.?

(764) In what time will the interest of £95 6*s.* 8*d.* amount to £12 17*s.* $4\frac{4}{5}$*d.*, at $4\frac{1}{2}$ per cent.?

(765) At what rate per cent. will £152 10*s.* amount to £191 7*s.* 9*d.* in 6 years?

(766) In what time will the interest of £135 15*s.* amount to £27 3*s.*, at 5 per cent.?

(767) In what time will £91 13*s.* 4*d.* amount to £100, at 3 per cent.?

Find the discount of

(768) £254 1*s.* 6*d.* for $2\frac{1}{3}$ years, at 3 per cent.

(769) £466 1*s.* 8*d.* for 6 years 8 months, at $3\frac{3}{4}$ per cent.

(770) £833 15*s.* for 3 years, at 5 per cent.

(771) £259 7*s.* for 4 years, at $3\frac{1}{2}$ per cent.

(772) A bill for £34 10*s.* drawn July 18, at 3 months, and discounted August 9, at $4\frac{1}{2}$ per cent.

(773) A bill for £14 9*s.*, drawn May 1, at 6 months, and discounted June 10, at 5 per cent.

(774) A bill for £923 2*s.* 7*d.*, drawn August 5, at 5 months, and discounted December 3, at $3\frac{1}{3}$ per cent.

(775) A bill for £1258 16*s.* 3*d.*, drawn April 30, at 90 days, and discounted June 22, at 3 per cent.

Find the present value of

(776) £1260 due 4 years hence, at 3 per cent.

(777) £2673 2*s.* 6*d.* due $3\frac{1}{2}$ years hence, at 5 per cent.

(778) £1196 19*s.* 6*d.* due 1 year 115 days hence, at $2\frac{1}{2}$ per cent.

(779) At what rate per cent. will the interest of £325 16*s.* 8*d.* amount to £48 9*s.* $4\frac{1}{4}$*d.* in $3\frac{1}{2}$ years?

(780) Find the simple interest of £1474 15*s.* 3*d.* from June 20 to November 18, at 6 per cent.

(781) Find the amount of £2700 for 4 years, at $3\frac{1}{2}$ per cent. compound interest, neglecting fractions of a penny.

(782) In what time will £2220 be doubled at 6 per cent. simple interest?

(783) Find the discount of £2267 0*s.* 7½*d.* payable 3¼ years hence, at 4 per cent.

(784) Find the amount of £550 for 4 years, at 5 per cent. compound interest.

(785) Find the discount of £1450 10*s.* paid 2 months before it was due, at 5 per cent.

(786) In what time will the interest of £1250 amount to £265 12*s.* 6*d.*, at 4¼ per cent.?

(787) What is the present value of £430, due 4 months hence, at 4 per cent.?

(788) A banker discounts a bill for £15 at two months, at 5 per cent.; how much will the holder receive? (Art. **23.**)

(789) Find the interest, at 3¾ per cent., on £924 12*s.* 8*d.*, lent March 18 and repaid August 11.

(790) What principal, lent at 3½ per cent. simple interest, will amount to £1884 19*s.* 10½*d.* in 5 years?

(791) What will be the amount, at compound interest, of £825, at the end of 2 years, at 4 per cent. per annum, the interest being due half-yearly?

(792) In what time will £171 7*s.* 6*d.* amount to £217 1*s.* 6*d.*, at 3⅓ per cent.?

(793) Find the discount of £501 10*s.*, due 6 months hence, at 4 per cent.

(794) Find the simple interest of £1049 16*s.* 6*d.* for 6 years and 4 months, at 4½ per cent.

(795) Find the amount of £3968 15*s.* in 4 years, at 3¼ per cent. simple interest.

(796) The interest of £11587 10*s.* for one year is £434 10*s.* 7½*d.*; find the rate per cent.

(797) Find the discount of £813 9*s.* for 1 year and 4 months, at 4¾ per cent.

(798) Find the present value of £1009 0*s.* 11¾*d.*, due September 24, discounted April 21, at 5 per cent.

(799) Find the present worth of a bill for £7712 0*s.* 4*d.*, due August 18 and discounted May 6, at 5 per cent.

(800) Find the discount of £2000 due 245 days hence, at 5½ per cent.

(801) Find the present worth of £82 5*s.*, due 5 years hence, at 3½ per cent.

(802) At what rate per cent. will £300 amount to £414 in 8 years?

(803) Find the amount of £875 for 15 months, at 4 per cent. per annum compound interest; the interest accruing quarterly.

(804) At what rate per cent. will £651 13*s.* 4*d.* amount to £748 12*s.* $0\frac{1}{2}$*d.* in $3\frac{1}{2}$ years?

(805) Find the discount of £126 13*s.* 4*d.* due 8 months hence, at 4 per cent. What would be the error in this case if interest were taken instead of discount, and in whose favour?

(806) Find the interest of £194, at 5 per cent., from March 3 to December 20.

(807) Find the present value of £275 6*s.* 8*d.*, due 15 months hence, at 4 per cent.

(808) Find the interest of £450 for half a year, at $4\frac{1}{2}$ per cent., allowing for income tax, at the rate of 7*d.* in the pound.

(809) Find the interest of £1158 17*s.* 6*d.* for 1 year 115 days, at $2\frac{1}{2}$ per cent.

(810) What sum of money put out to interest for 6 years, at $3\frac{1}{2}$ per cent., will amount to £259 7*s.* 5*d.*?

(811) At what rate per cent. will £951 17*s.* 6*d.* amount to £1142 5*s.* in 5 years?

(812) Find the discount of £594 14*s.* 9*d.*, due 8 months hence, at $5\frac{3}{4}$ per cent.

(813) What sum will amount to £45 0*s.* $9\frac{3}{4}$*d.* in a year, at $6\frac{1}{2}$ per cent.?

(814) At what rate will the interest of £4127 10*s.* amount to £92 17*s.* $4\frac{1}{2}$*d.* in a year?

(815) Find the compound interest of £364 for 3 years, at $3\frac{1}{4}$ per cent.

(816) Find the interest, at $4\frac{3}{4}$ per cent., of £1835 8*s.* 4*d.*, lent March 11 and repaid August 4.

(817) What sum of money will amount to £1482 11*s.* $10\frac{1}{2}$*d.* in 3 years, at $4\frac{1}{2}$ per cent.?

(818) At what rate per cent. will the interest of £1270 18*s.* 9*d.* amount to £76 5*s.* $1\frac{1}{2}$*d.* in two years?

(819) In what time will £1337 12*s.* 1*d.* amount to £1698 15*s.* $1\frac{3}{4}$*d.*, at 6 per cent.?

(820) Find the discount of £1511 7*s.* 6*d.* payable $3\frac{1}{4}$ years hence, at 4 per cent.

(821) Find the amount of £275 for 4 years, at 5 per cent. compound interest.

(822) Find the interest, at $4\frac{3}{4}$ per cent., of £1376 11*s.* 3*d.*, lent April 6 and repaid August 30.

(823) What sum of money will amount to £1281 12*s.* 1*d.* in 3 years, at $4\frac{1}{2}$ per cent.?

(824) At what rate per cent. will the interest of £635 9*s.* $4\frac{1}{2}$*d.* amount to £55 1*s.* $5\frac{3}{4}$*d.* in 2 years?

(825) In what time will £2674 4*s.* 2*d.* amount to £3396 4*s.* $10\frac{7}{10}$*d.* at 6 per cent.?

(826) Find the interest of £3000 for 8 months, at 4 per cent., less income tax, at 14*d.* in the pound.

(827) On what sum of money will the interest amount to £819 18*s.* $4\frac{39}{50}$*d.* in 4 years, at $3\frac{1}{2}$ per cent.?

(828) At what rate per cent. will £9999 19*s.* 9*d.* amount to £14999 19*s.* $7\frac{1}{2}$*d.* in 10 years?

(829) Find the discount of £512 15*s.* 3*d.* due 52 days hence, at $2\frac{1}{2}$*d.* per cent. per day.

(830) What must be the rate of interest in order that the discount of £1936 18*s.* due at the end of three years may be £207 10*s.* 6*d.*?

COMMISSION, BROKERAGE, INSURANCE.

(831) How much will a commission agent receive for selling goods to the value of £84 if he is allowed at the rate of $12\frac{1}{2}$ per cent.?

(832) How much will a broker receive for buying £2950 stock brokerage at £$\frac{1}{8}$ per cent.?

(833) What premium must be paid to insure goods worth £279, at 7 per cent., so that in the event of loss the premium paid may be recovered together with the value of the goods?

(834) A tradesman allows 10 per cent. discount for cash, what will be the cash price of an article marked 30*s.*?

(835) How much will the commission amount to on £187 16*s.* 10*d.*, at $2\frac{1}{2}$ per cent.?

(836) For what sum must goods worth £3700 be insured at $7\frac{1}{2}$ per cent., so that in the event of loss the worth of the goods and the premium paid may be recovered?

(837) Find the brokerage on the purchase of £8640 stock, at £$\frac{1}{8}$ per cent.

(838) A tailor's terms are 12 months' credit, or 5 per cent. discount for cash, how much will be saved by paying a bill of £19 10*s.* at once?

(839) What will be the commission on £20500, at 13*s.* 4*d.* per cent.?

(840) For how much must goods worth 7959 guineas be insured at $5\frac{1}{4}$ per cent., so that in the event of loss the value of the goods together with the premium paid may be recovered?

(841) What will be the annual payment to insure £750 on the life of a person aged 42; the annual premium for that age being £3 8*s.* 1*d.* per cent.?

(842) For what sum can the life of a person, aged 26, be insured by the annual payment of £35; the premium for that age being £2 6*s.* 8*d.* per cent.?

(843) A ship valued at £14500 is insured at £3 10*s.* per cent., and her cargo valued at £32000 is insured at 5 per cent.; find the whole cost of insurance.

(844) What premium at $2\frac{1}{2}$ per cent. must be paid to insure property worth £1170, so that in the event of loss the value of the property together with the premium paid may be recovered?

(845) Property which is worth £1710 and pays 7 per cent. is mortgaged to two-thirds its value at 4 per cent., what is the owner's net income?

STOCKS.

(846) How much 3 per cent. stock, at $91\frac{3}{4}$, can be bought for £1926 15*s.*?

(847) How much Great-Western stock, at $68\frac{1}{4}$, can be bought for £1351 7*s.*?

(848) A person sells £1000 Great-Northern stock at 127, how much money will he receive?

(849) How much will a person receive who sells £9990 out of the new 3 per cent. annuities at $91\frac{7}{8}$?

(850) What is the value of £1743 3 per cent. stock at $87\frac{7}{8}$?

(851) What is the cost of £650 bank annuities at $90\frac{3}{8}$ per cent., $\frac{1}{8}$ per cent. being paid for brokerage?

(852) How much $3\frac{1}{2}$ per cent. stock at $93\frac{7}{8}$ can be purchased for £2253, and what will be the half-yearly dividend?

(853) When the funds are at $81\frac{1}{4}$, how much stock can be bought for £3575?

(854) A person buys £500 stock at $98\frac{3}{4}$ and sells out at 103, how much does he gain?

(855) How much will be gained by investing £5000 in the 3 per cents. at $86\frac{1}{4}$ and selling out at $91\frac{1}{2}$?

(856) A person invests £333 6*s.* 8*d.* in the 3 per cents. at $88\frac{3}{8}$, what will be his half-yearly dividend?

(857) If I sell £13500 stock out of the $3\frac{1}{2}$ per cents. at par, and invest the proceeds in the four per cents. at $98\frac{1}{3}$, what difference shall I make in my income?

(858) What will be my yearly income if I invest £1700 in the 3 per cents. at $92\frac{1}{2}$?

(859) How much money must I invest in the $3\frac{1}{2}$ per cents. at 95 to produce an income of £700 a year?

(860) How much 4 per cent. stock can be purchased by the transfer of £1000 stock from the 3 per cents. at 72 to the 4 per cents. at 90?

(861) A broker invests £10830 at $90\frac{1}{8}$, and sells out on the funds rising to $90\frac{1}{4}$, how much does he gain?

(862) If I invest £1150 in the $4\frac{1}{2}$ per cents. at $95\frac{5}{8}$, what will be my half-yearly dividend?

(863) How much stock must be sold out of the $3\frac{1}{2}$ per cents. at $98\frac{1}{3}$ to pay a debt of £518 6*s.* 8*d.*?

(864) When the 3 per cents. are at £85 17*s.* 6*d.*, what is the rate of interest?

(865) A person who has £2954 stock in the 3 per cents. at $80\frac{1}{2}$ transfers his capital into the 4 per cents. at 98; find the alteration in his income.

(866) When the $3\frac{1}{4}$ per cents. are at $81\frac{1}{4}$, how much stock can I buy for £1300, and what rate of interest shall I get for my money?

(867) The price of consols is $88\frac{3}{8}$, how many railway shares, the market price of which is £8 15*s.*, can be bought for £1000 consols?

(868) A person sells as many 3 per cent. consols at $98\frac{5}{8}$ as produce £2000, and invests this sum in railway stock paying $4\frac{1}{4}$ per cent. at $93\frac{3}{4}$, how is his income affected?

(869) A person invests £5460 in the 3 per cents. at 91; he sells £2000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85; he then invests the produce in $4\frac{1}{2}$ per cents. at 102; what is the alteration in his income?

(870) A. invests £33075 in the purchase of 3 per cent. stock at $87\frac{1}{2}$, and B. invests the same sum in the purchase of 4 per cent. stock at $94\frac{1}{2}$; what will be the difference in their net incomes after deducting income tax at 16*d.* in the pound?

(871) What must be the price of 3 per cent. stock that by investing £32850 my income may be £1080 a year?

(872) What must be the market value of 6 per cent. stock in order that after deducting income tax at 10*d.* in the pound it may yield $6\frac{1}{2}$ per cent. interest?

(873) When a $3\frac{1}{4}$ per cent. stock is at 93, find what price a $4\frac{1}{2}$ per cent. stock must bear that an investment may be made with equal advantage in either stock.

(874) A person sells Midland stock, paying $6\frac{1}{2}$ per cent. at $128\frac{1}{2}$, and invests in Great-Western stock, paying 3 per cent. at $72\frac{1}{2}$, by how much per cent. will the interest of his investment be altered?

(875) A person invests £5000 in the new 6 per cent. Turkish loan,

issued at 68 per cent. at $2\frac{1}{2}$ premium; how much stock will he have, and what rate of interest will the investment give?

(876) What must be the market value of 3 per cent. stock in order that after deducting an income tax of 10*d.* in the pound it may yield $3\frac{1}{2}$ per cent. interest?

(877) Two persons have each a capital of £12000; one invests in the 3 per cent. consols at $90\frac{5}{8}$, the other in railway shares paying 5 per cent. at $103\frac{3}{4}$; find by how much the income of the one exceeds that of the other.

(878) A person sells £6000 3 per cent. consols at $92\frac{1}{4}$, and invests the proceeds in railway stock paying $5\frac{1}{2}$ per cent. at $103\frac{1}{8}$; find how much his income is affected (neglecting fractions of a penny).

(879) A person sells £8000 3 per cent. consols at 92, and invests the proceeds in railway shares paying 5 per cent. at 102, how much is his income increased?

(880) By a reduction of interest of Exchequer bills from $2\frac{1}{2}$*d.* to $1\frac{1}{2}$*d.* per cent. per day, a person loses at the rate of £152 7*s.* 9*d.* per annum; what amount of Exchequer bills does he hold?

(881) A person sells £5000 consols at $94\frac{7}{8}$, and on them rising sells £5000 more at $95\frac{5}{8}$; on their again rising he buys back the whole £1000 at 96; what does he lose?

(882) If the present price of 3 per cent. consols and Midland railway stock paying $5\frac{1}{4}$ per cent. are $95\frac{3}{8}$ and $108\frac{1}{2}$ respectively, compare the rates of interest which investments in those stocks would give.

(883) A person sells out of the 3 per cent. consols at 99, and invests in Exchequer bills bearing interest at the rate of $2\frac{1}{2}$*d.* a day per cent. when the bills are at a premium of 7*s.* 6*d.*, what effect has this on his income?

(884) A person bought 180 shares at the rate of 2 shares for £7, and 180 more at the rate of 3 for £7, after which he sold them all at the rate of 5 for £14, what did he gain or lose?

(885) A gentleman in Australia receives 12 per cent. per annum on his capital in the colony; he brings his capital home, invests in the 3 per cents. at $94\frac{5}{8}$, and his income in England is £2400 a year; what was his income in Australia?

PROFIT AND LOSS.

(886) An article which cost 22*s.* 6*d.* is sold for 27*s.*, what is the gain per cent.?

(887) At what price per lb. must butter which costs 10*d.* per lb. be sold so as to gain 15 per cent.?

(888) A horse which was bought for £45 is sold for £42 15*s.*, what is loss per cent.?

(889) By selling an article for 15*s.* 6*d.* I lose 7 per cent., what was the cost price?

(890) A 38-gallon cask of wine cost £25; 8 gallons are lost by leakage; at what price per gallon must the remainder be sold so as to gain 10 per cent. on the whole prime cost?

(891) A merchant having bought a 50-gallon cask of wine for £37 1*s.*, loses 3½ gallons. If 9 bottles make a gallon, at what price per dozen should he sell the remainder so as to gain 15 per cent.?

(892) If 5½ per cent. be gained by selling butter at £5 5*s.* 6*d.* per cwt., what will be the gain per cent. by selling it at 1*s.* 3*d.* per lb.?

(893) A stationer sold quills at 11*s.* a thousand, by which he cleared $\frac{3}{8}$ of the money, what would he gain per cent. by selling them at 13*s.* 6*d.* a thousand?

(894) Silk sold at the rate of 12*s.* 6*d.* per yard gained 7½ per cent., what did it cost per yard?

(895) By selling wine at 15*s.* a gallon 10 per cent. is lost, for what price per gallon must it be sold so as to gain 15 per cent.?

(896) If by selling goods for £272 15 per cent. is lost, how much would be gained or lost per cent. by selling them for 320 guineas?

(897) By selling an article for 5*s.* a person loses 5 per cent., for how much must he sell it so as to gain 4½ per cent.?

(898) An article which cost 7*s.*, and another which cost 11*s.*, are sold together for £1 4*s.* 9*d.*, what is the gain per cent.?

(899) Tea at 3*s.* 8*d.* per lb. is mixed with an equal quantity at 4*s.* 4*d.* per lb., for how much must the mixture be sold so as to gain 15 per cent. on the outlay?

(900) A farmer bought 27 oxen at £16 per head, and sold 18 of them for £20 per head, and the rest for £15 per head, how much did he gain per cent.?

(901) A person pays £30, including a duty of 5 per cent., for some French goods, how much was the duty?

(902) Hay bought at £4 16*s.* a ton is sold for £4 1*s.* 7½*d.*, how much is lost per cent.?

(903) For how much must goods which cost £11 18*s.* be sold so as to gain 12½ per cent.?

(904) How much did I give for 41 yards of cloth if I lose 18 per cent. by selling it at 4*s.* 7*d.* per yard?

(905) If I lose 7½ per cent. by selling goods for £2 6*s*. 3*d*., for what price must they be sold so as to gain 12½ per cent.?

(906) If I invest £1300 in a company whose shares are selling at 81¼ and which pays a dividend of 3¼ per cent. per annum, how much per cent. shall I get for my money?

(907) A dealer bought 4 horses for £125, and afterwards sold one fo £37, two for £35 each, and the other for £25, how much pe cent. did he gain by the transaction?

(908) If 23 lbs. at 2*s*. are mixed with 27 lbs. at 3*s*., and the mixture i sold at 2*s*. 9*d*. per lb., what is the total gain, and what is th gain per cent. on the money laid out?

(909) If 10 lbs. 3 oz. of tea at 4*s*. 8*d*. per lb. be mixed with 23 lbs. 3 o at 5*s*. 4*d*., and 17 lbs. 11 oz. at 6*s*. 8*d*., and the mixture be sold a 6*s*. 1*d*. per lb., what will be the gain or loss, and what the gai or loss per cent. on the cost price?

(910) By selling cloth at the rate of 12*s*. per yard 15 per cent. wa gained, how much per cent. would be gained by selling i at 10*s*. 6*d*. per yard?

(911) If 20 lb. 4 oz. of tea at 5*s*. 10*d*. be mixed with 46 lbs. 8 oz. a 6*s*. 8*d*. and the mixture be sold at 6*s*. per lb., what will be th gain or loss, and what the gain or loss per cent.?

(912) A person buys coffee at £5 12*s*. 6*d*. per cwt. and chicory a £2 5*s*. 5*d*. per cwt. and mixes them in the proportion of tw of chicory to five of coffee; he retails the mixture at 1*s*. 3*d*. pe lb., what is his gain per cent.?

(913) 6 per cent. is lost by selling rum at 15*s*. a gallon, at what pric should it be sold to gain 17½ per cent.?

(914) A grocer mixes tea worth 4*s*. 6*d*. per lb. and 3*s*. 6*d*. per lb respectively, in the proportion of 3 lbs. of the former to 2 lb of the latter, and sells the mixture at 4*s*. 4*d*. per lb., what doe he gain or lose per cent.?

(915) The expense attending the production of a book, the retail pric of which is 7*s*. 6*d*., is 2*s*. 4½*d*. per copy; the publisher allow the bookseller 25 per cent. on the retail price, and gives 1 copies to the dozen; 3900 copies are printed and sold. Th author is to have half the profits, how much will he receive?

(916) A merchant buys 15 dozen of port at 82*s*. per doz., and 60 do more at 50*s*. per doz. He mixes them and sells the mixture a 70*s*. per doz., what profit per cent. does he realise on hi outlay?

to sell them for 4*s.* apiece less than they cost him, at how much per head must he sell the remainder so as to gain on the whole $4\frac{17}{27}$ per cent. by the transaction?

PROPORTION PARTS.

(918) Divide £350496 amongst 4 persons whose shares shall be to each other in the ratio of 3, 7, 9, and 11.

(919) Divide 7395 into parts which shall have to each other the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$.

(920) A., B., and C. rent a pasture for £40; A. puts in 8 cows, B. 10, and C. 12, how much should each pay?

(921) A bank consisting of 4 partners is divided into 32 shares; A. holds 8, B. 9, and C. 10, and D. the remainder. They gain £5000, what is the share of each?

(922) A company of militia consisting of 108 men is to be raised from three towns, the population of which is 1500, 7000, and 9500 respectively, how many men must each furnish?

(923) A bankrupt, whose property realises only £112, owes three creditors £200, £150, and 200 guineas respectively, how much should each receive?

(924) Divide £3145 amongst 4 persons whose shares shall be to each other in the ratio of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{11}{12}$.

(925) A., B., and C. rent a field for £68 10*s.*; A. puts in 140 sheep for 14 days, B. 160 sheep for 12 days, and C. 200 sheep for 8 days, what should each pay?

(926) A bankrupt, whose estate realises £421 1*s.*, owes A. £256 6*s.* 8*d.*, B. £203 10*s.*, and C. £141 13*s.* 4*d.*, how much should they each receive?

(927) A. and B. entered into partnership; A. put in at first £2000, and at the end of 8 months £1000 more; B. put in at first £750, and at the end of 4 months £3000 more, but took £1300 out at the end of another 3 months; at the end of the year they had gained £1635, what should each receive?

(928) A ship worth £6000 is entirely wrecked; $\frac{1}{2}$ belonged to A., $\frac{1}{3}$ to B., and the rest to C., what are the respective losses supposing the ship to have been insured for £4500?

(929) Divide £20 amongst 6 persons, so that the share of each may be as much as those of all who came before put together.

(930) A bankrupt's estate which realises £21000 is to be divided amongst 4 creditors whose debts are A.'s to B.'s as 2 : 3, B.'s to C.'s as 4 : 5, C.'s to D.'s as 6 : 7, what must each receive?

(931) A loss of £149 is to be made good by 3 persons, A., B., and C. Had there been a gain, A. would have gained 4 times as much as B., and C. as much as A. and B. together, how much of the loss must each bear?

(932) A bankrupt owes £2085, of which £235 are due to A., £325 to B., £525 to C., and the rest to D.; how much must he pay in the pound that D. may receive as much as was owing to C., and what will his other creditors each receive?

(933) What weight of copper is there in a piece of metal weighing 2 tons 13 cwt. which is a mixture of tin and copper in the ratio of 9 of tin to 97 of copper?

(934) A tradesman starts with a capital of £960, and after 3 years takes another into partnership with £2100; after 4 years more the whole profits, £2304, are divided, how much should each receive?

(935) If the price of gold be 4 guineas per oz., what is the cost of a gold ornament weighing 3 oz., of which 18 parts out of 24 are pure gold, allowing 3*s.* 4*d.* per oz. for the value of alloy, and 25 per cent. on the whole for workmanship?

(936) A., B., and C. rent a house together for 2 years at £150 per annum; A. remains in it the whole time, B. 16 months, and C. 4½ months during the occupancy of B.; how much must each pay of the rent?

(937) Supposing in England gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal; in France of 77 parts of nitre, 9 of sulphur, and 14 of charcoal. If a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

EXCHANGE.

(938) When £170 will purchase 4233 French francs, what is the course of exchange between London and Paris?

(939) If London exchanges with Holland at a gain of 6¼ per cent. when the course of exchange is 35*s.* 6*d.* per £ sterling, what is the par of exchange?

(940) If £3 = 20 thalers, 25 thalers = 93 francs, 27 francs = 5 scudi, and 62 scudi = 135 gulden, how many gulden are equivalent to £1?

(941) If 20 piastres = £3⅜ and £1 = 25⅓ francs, how many francs are equivalent to 21 piastres?

what does he gain by sending it through France, the exchanges being £1 = 25·4 francs, 19 francs = 1 pistole, 4 pistoles = £3?

(943) How many milrees may be drawn at Lisbon for a bill of £284 6*s*. 8*d*. paid in London, exchange being at 3*s*. 10½*d*. per milree?

(944) A sum of £270 17*s*. 6*d*. is transmitted through Paris to New York; £1 = 24 francs 79 cents at Paris, and 6 francs 20 cents French = 2 dollars 25 cents American, what is the value of the sum at New York in American coinage?

(945) A person in London owes another at St. Petersburgh 920 roubles, which must be remitted through Paris; he pays the requisite sum to his broker at a time when the exchange between London and Paris is 25·15 francs for £1, and between Paris and St. Petersburgh 1·2 francs for 1 rouble. The remittance is delayed until the rates are 25·35 for £1, and 1·15 francs for 1 rouble, what does the broker gain or lose by the delay?

(946) Change 364 florins 10 stivers bank into current money. Agio 3½ per cent.

(16 pennings = 1 stiver, 20 stivers = 1 florin.)

(947) The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1, and the exchange at Milan upon Paris is at the rate of 42 Austrian lire for 20 francs, how many Austrian lire should be paid at Milan for £20?

(948) The exchange between London and Paris is 25·5 francs for £1, between Paris and Amsterdam 117 francs for 55 florins, between Amsterdam and Hamburgh 11 florins for 13 marks, what is the exchange between London and Hamburgh?

EQUATION OF PAYMENTS.

(949) Find the equated time of payment of £200 due 14 months hence, and £300 due 19 months hence.

(950) A. owes B. £1000, which is to be paid £100 at the end of one month, £200 more at the end of the next month, £300 more at the end of the next, and the remainder at the end of the fourth month; find the equated time of payment.

(951) £50 is due at the end of 2 months, £30 at the end of 3 months, and £60 at the end of 5 months; find the equated time of payment.

(952) Find the equated time of payment of £750, one-half of which is due in 4 months, three-eighths in 5 months, and the rest in 6 months.

(953) A debt of £1200 is to be paid at the end of a year; £200, however, is paid in a month, and £400 more in 4 months; when will the remainder be due?

(954) A debt which is due in $7\frac{1}{30}$ months is paid $\frac{1}{3}$ in 4 months, $\frac{1}{4}$ in 6 months, and $\frac{1}{5}$ in 8 months; when is the remainder due?

(955) A debt is to be discharged at the end of $4\frac{1}{2}$ months; $\frac{1}{4}$ is paid immediately, and $\frac{1}{4}$ at the end of 3 months; when ought the remainder to be paid?

SQUARE ROOT.

Find the square root of

(956) 361; 3025.
(957) 8836; 12321.
(958) 49729; 388129.
(959) 505521; 638401.
(960) 850084; 954529.
(961) 978121; 1234321.
(962) 131044; 2854·7649.
(963) 106929; 531441.
(964) 49660209; 379·8601.
(965) 36·060025; ·008281.
(966) 81·0016200081; 1195·50669121.
(967) ·0531118116; 4·93817284.
(968) ·00000182250; 5276491003812681.
(969) 10 and 7·1836, each to five places of decimals.
(970) 13 and 7·0032, each to four places of decimals.
(971) $32\frac{1}{9}$; $2515\frac{284}{361}$.

CUBE ROOT.

Find the cube roots of

(972) 29791; 54872.
(973) 3375; 804357.
(974) 1092727; 4085184.
(975) 586376253; 946966163.
(976) 1194389981; ·241804367.
(977) 134217728; 11089·567.
(978) 932·574833; 1006·012008.
(979) ·000057464; ·000405224.
(980) 2 and 25, each to three places of decimals.
(981) 5·78 and ·05, each to three places of decimals.

AVERAGES AND PERCENTAGES.

(982) Find the average of $17\frac{1}{2}$, $25\frac{1}{4}$, $96\frac{3}{8}$, 10, 0, $42\frac{3}{4}$, 56.

(983) In a certain office there is one person receiving £2000, two who receive £1100, six who receive £400, twelve who receive £200, and eighteen who receive £90 a year each; what is the average income of the persons employed in the office?

(984) Find the average of $21\frac{2}{3}$, $73\frac{4}{5}$, 0, 3·065, 82, $17\frac{3}{20}$, $5\frac{1}{4}$, $9\frac{5}{12}$, and express the fractional part decimally

(985) The population of a place at one period was at 4780 and another period was 5002; what was the increase per cent.?

(986) Of 23 selected candidates for the East India Civil Service in 1859, 3 were above 20 years of age when they went to India, 4 above 21, 12 above 22 and 23 respectively, and 1 above 24. From these data find what was the average age at which the men went to India.

(987) Find the average of $13\frac{12}{25}$, 21, $7\frac{3}{4}$, ·0023, $3\frac{1}{8}$, 0, $106\frac{1}{2}$, and $57\frac{7}{20}$, and express the fractional part decimally.

(988) The population of Great Britain in 1851 was 21,121,957. The increase during the previous half century had been 95·5 per cent.; what was the population in 1801?

(989) A person's average annual expenditure from the year 1833 to 1853 inclusive, is £293 17*s.* 4*d.* He finds that in 1833 he spent £283 17*s.* 10*d.*, and in 1854, £365 14*s.* 1*d.*; what was his average annual expenditure from 1834 to 1854 inclusive?

(990) Between the years 1841 and 1851 the population of England increased 14·2 per cent. In the latter year it was 21,121,290. What was it in the former year?

(991) If the Roman Catholics are 3 to 1 of the population of Ireland, and the Protestant Dissenters bear the proportion of 2 to 3 to the members of the Established Church, find the proportion per cent. which the Protestant Dissenters bear to the Roman Catholics.

(992) A tradesman's annual losses during 5 years average $1\frac{1}{4}$ per cent. on the capital with which he began, and at the end of the five years his effects are worth £2531 5*s.* What capital did he begin with?

(993) An army lost 18 per cent. of its strength by disease and desertion, and then lost 14 per cent. of the remainder in battle. The number then remaining was 84624; of how many did the army originally consist?

(994) In 1841 the population of Great Britain was 21476000, and that of Ireland was 7310000. In 1851 the former had increased 8·45 per cent. and the latter had decreased 12·5 per cent. Find the increase per cent. in the population of the whole kingdom.

(995) Sugar being composed of 49·856 per cent. of oxygen, 43·265 per cent. of carbon, and the remainder hydrogen; find how many pounds of each of these materials there are in 1 cwt. of sugar.

(995) The populations of 3 towns in the year 1841 were 15875, 64575, and 29185. In the year 1851 the two former had increased

respectively 4 and 8 per cent., and the latter had decreased 20 per cent. Find the increase or decrease per cent. of the total population of the three towns.

(997) In 1858 the value of £100 tithe rent-charge reckoned on the average price of corn in the 7 years immediately preceding was £105; in 1859, reckoned in the same way, it is 3 per cent. more. If it were reckoned on the price in 1851 only, it would be but £69. What would it be if reckoned on the price in 1858 only?

(998) The populations of 3 towns in the year 1841 were 20325, 41304, and 6117; in the year 1851 they had increased respectively 9, 10, and 12 per cent. Find the average population of the three towns in 1851.

(999) The population of five parishes being 1236, 452, 364, and 516 and 3430 respectively; find what the population of a sixth parish must be in order that the average population of the six may be 1256·5.

(1000) The number of railway passengers during the same period in England, France, and Prussia respectively, were—1st class, 4743213, 2124917, and 307492; 2nd class, 10291749, 6172429, and 1292476; 3rd class, 21409217, 15473551, 2374237. Find the proportion per cent. which the whole number of 1st class passengers bears to the whole number of 2nd and 3rd class passengers.

(1001) An electric cable is laid between two ports, and it is found that the gross annual receipts amount to 20 per cent. on the original outlay; the annual cost of repairs is 2 per cent. on the original cost, and the expense of working is found to be £600 a year. in addition to 20 per cent. on the net receipts. After a year, a second cable is laid down, and it is found that the gross receipts and the expense of working are doubled, while the cost of repairs for the new cable is only 1 per cent. and the net receipts now amount to £14500. Find the cost of a cable and the net receipts for the first year.

MISCELLANEOUS.

(1002) If £1 8 fl. 3 c. 7½ m. be paid for 7 bushels, how much must be paid for 3 bushels?

(1003) A square tower is 97 ft. high, and its solid content 5464⅓ c. yds. *Find its* breadth, and how much ground it stands on.

(1004) What will be the cost of excavating a well whose diameter is 3 ft. 9 in. and depth 42 yds. 2 ft., at 8*s.* 9*d.* per c. yd.?

(1005) If a man rows 10 miles in 2 hours and a half, against a stream the rate of which is 3 miles an hour, how long would he be rowing 5 miles with the stream?

(1006) A bankrupt's assets amount to £179 2 fl. and his debts are £405 2 fl.; how much can he pay in the pound?

(1007) Find the simple interest of £1050 6 fl. 2 c. 5 m. for 6 years, at $4\frac{1}{4}$ per cent.

(1008) How many gun-cards, each half an inch in diameter, can be punched out of a piece of cardboard 1 ft. square?

(1009) How many yards of muslin, ·75 yd. wide, will be required to line 55·75 yds. of silk ·875 yd. wide?

(1010) The length of one degree being $69\frac{1}{22}$ miles, what is the length of the diameter of the earth, supposing the ratio of the diameter of a circle to the circumference is 7 : 22?

(1011) A bankrupt pays 5 fl. 7 c. 5 m. in the pound; how much will a creditor lose to whom he owes £11793 5 fl.?

(1012) The side of a cube is 6 s. ft. 97 s. in.; find the solid content.

(1013) A bankrupt has good debts to the amount of £456 18*s.* 1*d.* and the following bad debts: £360 7*s.* 10*d.*, £120 13*s.*, and £19 18*s.*, for which he receives respectively 4. 5, and 9 shillings in the pound; his own liabilities amount to £3408 12*s.* How much can he pay in the pound?

(1014) The rate of a clock is ·0375 per cent. too fast; how much will it gain in a week?

(1015) If 11 cwt. can be carried 12 miles for £1 5c., how far can 36 cwt. 23 lbs. be carried for £5 2 fl. 5c.?

(1016) What number is that whose square divided by the fourth root of $150\frac{1}{16}$ is equal to 14?

(1017) If 12 bags of potatoes of 2 bushels each, at 3*s.* 4*d.* per bushel, last 8 persons 3 months, what will be the price per bushel when 9 bags of 3 bushels each last 6 persons $\frac{1}{3}$rd as long again?

(1018) If water flows through a pipe $2\frac{1}{2}$ in. in diameter at the rate of 1 ft. per sec., how many gallons will be discharged in an hour?

(1019) A reservoir is 24 ft. 8 in. long, and 12 ft. 9 in. wide; how many cubic feet of water must be drawn off to make the surface sink one inch?

(1020) In what proportion must tea which cost 6*s.* per lb be mixed with some at 4*s.* per lb. so that 20 per cent. may be gained by selling the mixture at 5*s.* 3*d.* per lb.?

(1021) Find the discount on £775 6 fl. 1 c. $2\frac{1}{3}$ m. due 5 months hence, at $4\frac{1}{2}$ per cent.

(1022) A person travelling on the Great Western Railway observes that he is 21 seconds in passing 24 telegraph posts, which he knows are 25 yds. 2 ft. apart. When passing a station, the speed is reduced one-third, and he finds they are $22\frac{1}{2}$ seconds going through; what is the length of the station?

(1023) How much water must be added to a pipe of wine which cost £90, to reduce its price to 10*s.* a gallon?

(1024) If 13 horses, *or* 17 cows, *or* 29 sheep, can graze a field of 7 acres in 11 days, in how many days will 11 horses, 22 cows, and 33 sheep graze 1337 acres?

(1025) A gallon of water weighs 10lbs. Find the pressure of the atmosphere on a square inch when a water barometer stands at 33 ft.

(1026) A man having worked 30 days, finds he has done only $\frac{3}{5}$ths of his work; he then calls in another to help him, and they finish it in 6 days; in what time would each do it separately?

(1027) The edge of a cube is 18 in.; what is the edge of a cube 8 times as large?

(1028) Find the time when the hour and minute hands of a watch are together between 2 and 3 o'clock.

(1029) A down train, which travels 36 miles an hour, leaves Paddington at the same instant that an up-train, which travels 45 miles an hour, leaves Didcot, 54 miles from London; at what distance from London will they pass each other?

(1030) If 7 horses *or* 23 sheep can graze a field of 10 acres in 7 days, in how long will 23 horses and 23 sheep graze 60 acres?

(1031) A., B., and C. can do a piece of work in 8, 9, and 12 days respectively. A. works at it alone for 3 days, and is then joined by B., and they work together for $1\frac{1}{2}$ day; A. then leaves and C. comes on; when will the work be finished?

(1032) Express an English mile in French metres, a metre being equal to 39·371 English inches.

(1033) The lengh of the seconds pendulum at Greenwich is 39·1393 in., and the force of gravity is measured by the product of (the length of the seconds pendulum) $\times (3{\cdot}14159)^2$. Find the expression for the force of gravity in feet.

(1034) A hare pursued by a dog has a start of 104 yds., and the dog runs 8 yds. for every 6 that the hare runs; how far will the dog have to run before he catches the hare?

(1035) A shilling weighs 3 dwts. 15 grs., of which 3 parts out of 40 are alloy, and the rest pure silver. How much per cent. is there of alloy, and what is the weight of pure silver?

6) A ship is wrecked whose value is £20000; $\frac{1}{3}$rd belonged to A., $\frac{1}{4}$th to B., $\frac{1}{5}$th to C., and the rest to D. The insurance office pays them £12000; how much should each receive?

7) At what rate per cent. will a given sum double itself in 25 years at simple interest?

8) In what time will a given sum treble itself at 5 per cent. simple interest?

9) Find the time between 8 and 9 o'clock, when the hour and minute hands of a watch are (1) exactly opposite, (2) at right angles, (3) coincident.

0) A man completes $\frac{2}{7}$ths of a piece of work in 13 days; he then receives the assistance of another, and they finish it together in 6 days; in what time would each do it separately?

1) If 12 oxen are worth 29 sheep; 15 sheep worth 25 hogs; 17 hogs worth 3 loads of wheat; and 8 loads of wheat worth 13 loads of barley; how many loads of barley must be given for 20 oxen?

2) A cistern can be filled by two pipes in 2 and 3 hours respectively, and emptied by another in an hour and a half; how long will it be filling if they are all open together?

3) I lay out £1000 in the purchase of £5 shares in a mine when they are at 2 premium, and £500 more in the purchase of £2 shares in another mine when they are at $\frac{3}{4}$ discount. The shares in the first fall to par, but those in the second rise to 3 premium, and I then sell out of both. How much do I gain after paying the broker $\frac{1}{4}$ per cent. on all the money which passes through his hands?

4) Two clocks point to 12 o'clock at the same instant; one loses 7 seconds and the other gains 8 seconds in 24 hours. When will one be half an hour before the other, and what time will each clock then show?

5) The rate of a clock is ·04 per cent. too fast. It is put right at 9 A.M. What time will it show at 3 P.M.?

6) In the month of December the number of paupers in a certain union was 336, the number of women being double that of the men, and the children being as many as the men and women together. If a man cost one-third more than a woman, and three children as much as a man and a woman, and the whole cost for the month be £83 6*s*., how much is the daily cost of each?

(1047) A steamer makes the passage from New York to Liverpool (2760 miles) in 9 days 14 hours, and a train goes from London to Edinburgh (405 miles) in 18 hours; compare the rates of the steamer and the train.

(1048) A vessel whose speed was $9\frac{1}{4}$ miles an hour, started at 8 A.M. to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 : 5, starting from the same place, arrived 5 minutes before the first. When did the second vessel start?

(1049) At a siege it was found that a certain length of trench could be dug by the soldiers and navvies in four days; but when only half the navvies were present, it required seven days to dig the same length of trench; what proportion of the work was done by the soldiers?

(1050) Find the cost of covering a table $6{\cdot}4\dot{2}8571\dot{4}$ yards long, and $1{\cdot}7\dot{8}$ yards wide, at 25*s.* the square inch.

(1051) A. had 12 pipes of wine which he sold to B. at $4\frac{1}{2}$ per cent. profit; B. sold them to C. for £40 12*s.* profit; and C. sold them to D. for £605 18*s.* and cleared 6 per cent. What did A. give a gallon for the wine?

(1052) The proportion of unpaid letters to the whole number posted in London was 8 per cent., and of the paid letters 51 per cent. were stamped, and the whole number posted was 450,000; how many of them were stamped?

(1053) A ship 40 miles from shore springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes, and 60 tons would be sufficient to sink her; but the ship's pumps can throw out 12 tons in an hour. Find her rate of sailing so that she may reach the shore just as she is beginning to sink.

(1054) The capital of a mining company is £300,000; the working expenses of the mine amount to £15691 13*s.* 4*d.* in a year; what must be the gross proceeds of the mine to pay the original proprietors 4 per cent.? Also, supposing the original shares to be £10, what percentage will a person receive who purchased at 2 discount?

(1055) Having borrowed £220 at 5 per cent.; in 12 months after I borrowed £340 more at 4 per cent. It took £607 18*s.* to discharge both debts. When was the money repaid?

(1056) A traveller sets out to walk from A. to B., at the rate of 4 miles an hour. An hour later another traveller sets out from B. towards A. at $3\frac{1}{2}$ miles an hour. When they meet the first has walked 6 miles more than the second. How far is it from A. to B.?

QUESTIONS.

1. DEFINE 'a Unit,' 'Number,' 'Five,' and distinguish between Notation and Numeration.

2. How many different symbols or figures do we require to express all numbers by the ordinary system of notation? What are they? Why is it called the 'Arabic System of Notation?'

3. Explain fully the ordinary system of notation. What do you mean by 'Local Value?' What system of notation did the Romans use?

4. Distinguish between 'Abstract' and 'Concrete' numbers.

5. Define 'Multiplicand,' 'Multiplier,' 'Product,' and 'Continued Product.'

6. What do you mean by a 'Power' of a number? How is it expressed? Define the terms Square Root and Cube Root.

7. What is a 'Factor?' Distinguish between 'Prime' and 'Composite' numbers.

8. Show that a number will be divisible by 8 when its three last digits are divisible by 8, and by 9 when the *sum* of its digits is divisible by 9.

9. How would you prove the correctness of a sum (1) in multiplication, (2) in division, by 'casting out nines.'

10. Define the terms 'Greatest Common Measure' and 'Least Common Multiple.'

11. Define a 'Fraction.' Which of its parts is the Numerator, which the Denominator? Why are they so called?

12. Define a 'Proper Fraction,' an 'Improper Fraction,' a 'Mixed Number,' a 'Compound Fraction,' and a 'Complex Fraction,' and show that an improper fraction is equivalent to a mixed number, and vice versâ.

13. Show that the value of a fraction will not be altered if the numerator and denominator be both multiplied or both divided by the same number; and hence show how an integer may be expressed in the form of a fraction having any given number for its denominator.

14. When is a fraction in its 'Lowest Terms?'

15. Explain the rule for reducing fractions to equivalent ones having a common denominator. In adding and subtracting fractions why is it necessary to reduce them to a common denominator?

16. Explain why the rule for reducing a compound fraction to a simple one is the same as that for multiplication of fractions.

17. Show how the ordinary system of notation may be extended to fractions.

18. Define a 'Decimal.' What is the 'Decimal Point?' What advantages do decimal fractions possess over vulgar fractions?

19. From the definition of a decimal deduce the rules for the multiplication and division of decimals.

20. Explain the rule for reducing a vulgar fraction to a decimal. Can all vulgar fractions be expressed exactly as decimals?

21. How can you tell whether a vulgar fraction will produce a finite decimal or not? Show that if it does not produce a finite decimal it *must* produce a recurring one.

22. In what sense is a recurring decimal said to be equal to a vulgar fraction? What is the greatest number of figures there can be in the period of a recurring decimal?

23. What is the standard (1) of the gold, (2) of the silver coin of this country?

24. Explain fully how the standards of the weights and measures are determined in England and in France respectively. What is the French system called?

25. What advantages should we derive from a decimal coinage?

26. What is a 'Mean Solar Day?' What do you mean by 'Leap Year?' Who first introduced it? and why? What alteration has since been found necessary? By whom was it made? How would you ascertain whether any given year is leap year or not?

27. What do you mean by the 'Old Style' and 'New Style?'

28. What must we be careful to observe in reducing poles and perches to yards, and vice versâ?

29. When dividing by the factors of a composite number, how do you form the true remainder?

30. Explain fully the principle of 'borrowing and carrying' in subtraction.

31. To what class of examples is the method of 'Practice' applicable? What is an 'Aliquot Part?'

32. Define Proportion and Ratio. How is it that if we know three terms of a proportion we can find the fourth? Must the two terms of a ratio be of the same kind? What does a ratio always represent, whatever its terms may be?

33. Deduce the 'Rule of Three' from the principles of proportion.

34. Define the terms 'Area' and 'Solid Content.' Show how to find the area of a rectangle and the solid content of a rectangular solid.

35. Having given the area and one dimension, or the solid content and two dimensions, how would you find the other dimension?

36. What is the difference between Simple and Compound Interest? And what do you mean by the 'Principal,' 'Rate per cent.,' and 'Amount?'

37. Distinguish clearly between Interest and Discount, and define Discount and Present Worth.

38. How is discount calculated in practice, and which party gains the advantage? What do you mean by 'Days of Grace?'

39. What do you mean by the terms Commission, Brokerage, Insurance, Premium, and Policy of Insurance?

40. Define 'Stock' and explain why the value of stock varies. When is stock said to be at par? at a premium? at a discount?

41. Explain the terms 'Dividend,' 'Funds,' 'Fundholder.'

42. Define 'Exchange,' 'Course of Exchange,' 'Par of Exchange,' 'Agio,' 'Usance,' and 'Arbitration of Exchanges.'

43. Explain briefly why the course of exchange between two countries varies. When is the exchange said to be against a country?

The bills of exchange drawn by one country on another, as, for instance, by England on France, will be of more or less value according as there is more money due at the time by England to France, or the contrary; and if the English merchants have claims on the French merchants to a greater amount than the French merchants have upon them, bills drawn in France upon England will be worth a little more than the amount stated. The rate of exchange will then be said to be against France.

44. Explain the reason for 'pointing' in the extraction of the square root.

Since $1=1^2 : 100=10^2$, the square root of every number between 1 and 100, i. e. of every number having *one* or *two* digits lies between 1 and 10, and is therefore a number of *one* digit. Again, since $100=10^2 : 10000=100^2$, the square root of every number between 100 and 10000, i. e. of every number having *three* or *four* digits, lies between 10 and 100, and is therefore a number of *two* digits. Similarly, the square root of every number of *five* or *six* digits is a number of *three* digits, and so on. If therefore a point be placed over the unit's place, and then over every alternate digit to the left, the number of points will indicate the number of digits in the root. This is one use of pointing.

Again, the number whose root is to be extracted is thus divided into periods, each of which corresponds to a digit in the root, and may be taken separately; for the first figure in the root is evidently the greatest number whose square is not greater than the first period, and therefore the periods after the first may be neglected in finding this digit. Similarly, the two first digits of the root form the greatest number whose square is not greater than the two first periods, and therefore the periods after the second may be neglected in finding the second digit in the root. That is, after finding the first digit from the first period, the second period only need be brought down to determine the second digit, and so on.

The reason for pointing in the extraction of the cube root may be explained in a similar manner.

ANSWERS.

GREATEST COMMON MEASURE.

(1) 135. 513.   (2) 13. 37.   (3) 99. 1.
(4) 43. 53.   (5) 2664. 1003.

LEAST COMMON MULTIPLE.

(6) 240.   (7) 1440.   (8) 72.   (9) 2520.
(10) 22680.

VULGAR FRACTIONS.

(11) $\frac{7}{8}, \frac{35}{24}, \frac{7}{4}, \frac{49}{24}, \frac{7}{3}, \frac{77}{24}$.   (12) $\frac{4}{13}, \frac{20}{39}, \frac{40}{39}, \frac{4}{3}, \frac{56}{39}$.
(13) $\frac{4}{13}, \frac{3}{13}, \frac{12}{65}, \frac{3}{13}, \frac{12}{91}$.   (14) $\frac{3}{19}, \frac{2}{19}, \frac{18}{190}, \frac{18}{209}, \frac{1}{19}$.
(15) $\frac{14}{2}, \frac{21}{3}, \frac{28}{4}, \frac{35}{5}, \frac{42}{6}$.   (16) $\frac{56}{7}, \frac{64}{8}, \frac{72}{9}, \frac{80}{10}, \frac{88}{11}$.
(17) $\frac{38}{7}, \frac{135}{13}, \frac{461}{40}$.   (18) $\frac{253}{20}, \frac{63}{13}, \frac{464}{27}$.
(19) $\frac{803}{83}, \frac{2497}{119}, \frac{10301}{101}$.   (20) $4\frac{5}{8}, 3\frac{5}{6}, 3\frac{1}{16}$.
(21) $11\frac{5}{12}, 12\frac{1}{13}, 11\frac{6}{17}$.   (22) $12\frac{148}{155}, 2\frac{70}{117}, 10010\frac{991}{1001}$.
(23) $\frac{9}{11}, \frac{5}{6}, \frac{2}{3}$.   (24) $\frac{1}{9}, \frac{20}{21}, \frac{143}{98}$.   (25) $\frac{6}{7}, \frac{17}{19}, \frac{3}{4}$.
(26) $\frac{117}{119}, \frac{7}{8}, \frac{5355}{7144}$.   (27) $\frac{3}{29}, \frac{17}{20}, \frac{3}{4}$.   (28) $\frac{79}{85}, \frac{121}{456}, \frac{9}{11}$.
(29) $\frac{99991}{97073}, \frac{1785}{3572}, \frac{73}{189}$.   (30) $\frac{1}{2}, \frac{2}{5}$.   (31) 4, 1.
(32) $40\frac{1}{2}, 2\frac{5}{8}$.   (33) $\frac{14}{15}$.   (34) $290\frac{19}{22}$.
5) $\frac{28,\ 90,\ 128,\ 129,\ 135}{144}, \frac{480,\ 616,\ 805,\ 888,\ 180}{840}$.
6) $\frac{48552,\ 21420,\ 12600,\ 6528,\ 22491}{51408}, \frac{2380,\ 2520,\ 1323,\ 442,\ 3060}{4284}$.
7) $2\frac{7}{9}, 2\frac{29}{105}$.   (38) $1\frac{17}{80}, 2\frac{43}{60}$.   (39) $8\frac{11}{18}, 13\frac{61}{120}$.
0) $16\frac{5}{16}, 12\frac{7}{48}$.   (41) $15\frac{3}{7}, 15\frac{1}{12}$.   (42) $5\frac{31}{40}, 5\frac{89}{120}$.
3) $10\frac{101}{126}, 5\frac{22}{35}$.   (44) $11\frac{389}{2400}, 9\frac{11}{12}$.   (45) $2\frac{7}{15}$.
6) $21\frac{11}{12}$.   (47) $\frac{1}{8}, \frac{13}{24}, 1\frac{7}{18}$.   (48) $1\frac{1}{8}, 2\frac{22}{35}, \frac{55}{63}$.
9) $10\frac{7}{8}, 12\frac{35}{36}, 4\frac{31}{120}$.   (50) $3\frac{35}{36}, \frac{47}{60}, 2\frac{37}{48}$.   (51) $4\frac{17}{18}, 3\frac{3}{4}, 3\frac{17}{44}$.
2) $\frac{1}{42}$.   (53) $1\frac{13}{24}$.   (54) $7\frac{7}{24}$.
5) $11\frac{23}{24}$.   (56) $\frac{49}{60}$.   (57) $1\frac{3}{8}$
) $\frac{5}{16}, \frac{2}{11}$.   (59) $15\frac{7}{9}$, 63, 56.   (60) 777, $97\frac{3}{8}$, 84.

(61) $39, 35, 34\frac{49}{264}$.
(62) $\frac{70}{221}$, 32, 81.
(63) $\frac{7}{8}, \frac{1}{2}, 1\frac{2}{5}$.
(64) $2\frac{1}{16}, \frac{87}{115}, 1\frac{38}{43}$.
(65) $\frac{34}{65}, 4\frac{20}{49}, \frac{1}{5}$.
(66) $10, \frac{7}{44}, 2\frac{1}{2}$.
(67) $1\frac{43}{280}, \frac{25}{51}$, 4.
(68) 3, 1.
(69) 1.
(70) 1.
(71) 185.
(72) 1.
(73) $\frac{52}{225}$.
(74) $1\frac{533}{2745}, 3\frac{6523}{7686}$.
(75) $\frac{11}{2400}$.
(76) $67\frac{1}{5}$.
(77) $\frac{25}{48}$, 1.
(78) $\frac{14}{45}$, 2.
(79) $\frac{1}{2}$.
(80) 1.
(81) $3\frac{11}{90}$.
(82) $1\frac{1}{7}$.
(83) $15\frac{5}{6}$.
(84) $\frac{1}{24}$.
(85) $25\frac{85}{98}$.
(86) $\frac{247}{1224}$.
(87) $\frac{1}{26}$.
(88) 1.
(89) $\frac{493}{504}$.
(90) $\frac{1679}{1686}$.
(91) $1\frac{4}{7}$.
(92) 2.
(93) 1.
(94) 2.
(95) $\frac{17}{52}$.
(96) $1\frac{17}{228}$.
(97) 1.
(98) $\frac{551}{1098}$.
(99) $3\frac{9}{11}$.
(100) $\frac{1668}{23903}$.
(101) $\frac{3}{8}$.
(102) $\frac{38}{45}$.
(103) $1\frac{14}{17}, \frac{17}{25}$.
(104) $\frac{4}{15}$.
(105) $\frac{1}{7}$.

DECIMAL FRACTIONS.

(106) 51·5675.
(107) 428·9573.
(108) 766·599.
(109) 334·3603.
(110) 262·027571.
(111) 12988·76693.
(112) 51206·082.
(113) 2000·83672.
(114) 843·8985.
(115) 1002·46165.
(116) 486·622, 6·2436.
(117) 33·06886, 2·109.
(118) 118·114, ·30019.
(119) 5·160023, 21·50101.
(120) 2·3884, 43·04922.
(121) 446·352, 55·0124.
(122) 865·49892, ·3864.
(123) ·907905, 406·1891.
(124) ·9999.
(125) 1·0001
(126) 2·47975, 636·83564.
(127) ·035594, 2·691066.
(128) ·2460168, 140·99575152.
(129) 5424·8324832. 3·12834
(130) 11961·232, 318·906015.
(131) 725·70045, 112·9115.
(132) 31·40399616, 158466·011.
(133) 2·6991408, ·019683.
(134) 2000·983462491.
(135) 267, 237286.
(136) 1000, 6.
(137) 130, 13, 1·3.
(138) 90·08, 450400.
(139) 245·53, &c., 9640.
(140) 6·25, ·0384759375.
(141) 37·56, 1200.
(142) ·08, 400, 39·7543782, &c.
(143) ·04068.
(144) ·308.
(145) 1·137.
(146) ·0008
(147) 1300, ·13, 13000, ·011, ·0011, 110.
(148) ·00625.
(149) 118·8.
(150) Sum = ·00027; diff. = ·00001; product = ·000000182.
(151) 16·01168.
(152) 9·3384 days.

2·8, ·1256, 13·34, ·5625. (158) ·$\dot{6}$, $\dot{8}5714\dot{2}$, ·$\dot{9}\dot{0}$.

·$\dot{0}7692\dot{3}$, ·7$\dot{3}$, ·6$\dot{9}$. (160) ·$\dot{5}2380\dot{9}$, ·13$\dot{0}\dot{9}$, ·90$\dot{6}$75.

·04, ·06, ·015625. (162) 13·96478.

·232$\dot{1}4285\dot{7}$, ·98. (164) $\frac{33}{3125}$, $23\frac{61}{1600}$, $\frac{11}{1600}$.

$\frac{1}{80}$, $2\frac{89}{2000}$, $\frac{1}{1600}$. (166) $\frac{7}{4000}$, $\frac{13}{4000}$, $\frac{8}{625}$.

$3\frac{181}{2500}$, $\frac{1}{40}$, $\frac{3057}{31250}$. (168) $\frac{1}{3}$, $\frac{1}{30}$, $\frac{805}{1111}$.

$\frac{349}{2200}$, $\frac{17}{330}$, $\frac{67}{495}$. (170) $\frac{17}{1375}$, $\frac{13}{90}$, $\frac{8546}{24975}$.

$11\frac{19}{66}$, $\frac{36}{275}$, $\frac{6997}{18500}$. (172) $\frac{11123}{18500}$, $\frac{17}{65}$, $9\frac{209}{225}$.

10·81824. (174) 39 93562.

105·05782. (176) 18·32373.

1·86464, 1·04213. (178) 17·76867, ·93213.

·307988, &c., ·012$\dot{5}1\dot{8}$, 61·$\dot{1}6\dot{5}$. (180) ·$\dot{9}0476\dot{1}$, 1·5, 1·3$\dot{6}$.

3·$\dot{6}$, 1·1$\dot{4}\dot{5}$. (182) ·5$\dot{2}$, 12.

1·5, ·0000437, &c. (184) 33·3, 308·34.

1·$\dot{2}$. (186) ·70076, &c.

1·5488. (188) 13·0125. (189) ·0642.

$1\frac{155}{256}$, 1·60546875. (192) 1·0$\dot{7}1428\dot{5}$.

33·09016. (194) 540000, ·0054. (195) 8585.

1250, 12500, ·000125. (197) $11\frac{139}{270}$, 11·5$\dot{1}4\dot{8}$.

35·4875. (199) ·1925. (200) 1·25.

REDUCTION.

506722, 188919. (202) 9599, 725491.

591948, 1170360. (204) 4193280.

111510. (206) 1010647. (207) 582205.

3361968. (209) 23061. (210) 629579.

562832. (212) 673328. (213) 1358880.

1820752. (215) 1574624. (216) 838667.

8076668. (218) 175350. (219) 101095.

375057. (221) 57367. (222) 2303037.

22188. (224) 99528. (225) 91214.

78256. (227) 85931. (228) 266450.

79573. (230) 554. (231) 4169.

160757. (233) 26680. (234) 58191.

865170. (236) 2703294. (237) 1518900.

11114940. (239) $495611\frac{1}{2}$. (240) 48516.

192669. (242) 49602. (243) 869460.

28455. (245) 17587. (246) 136240.

543104. (248) $221339\frac{1}{4}$. (249) 111806373.

5751989. (251) 96329124. (252) 615187.

488245. (254) 94153. (255) 9044.

(256) 133056. (257) 40704. **(258) 943197.**
(259) 4555380. (260) 3055709.

---

(261) £439 3*s.* 2½*d.*, £3874 9*s.* 1¼*d.*
(262) 7763 gui. 17*s.* 10¾*d.*, 2436 moi. 25*s.* 9*d.*
(263) 8933 gui. 5*s.* 6*d.*, 57635 crs. 1*s.*
(264) 548 cwt. 3 qrs. 25 lbs. 7 oz., 111 qrs. 4 lbs. 10 oz. 3 drs.
(265) 115 tons 17 cwt. 2 qrs. 6 lbs., 428 tons 4 cwt. 13 lbs. 11 oz.
(266) 2751 cwt. 3 qrs. 24 lbs. 9 oz. 1 dr., 11 tons 19 cwt. 20 lbs. 8 oz. 1dr.
(267) 4 lbs. 11 oz. 16 dwts. 22 grs., 3328 lbs. 2 oz. 5 dwts.
(268) 156 lbs. 3 oz. 3 dwts. 3 grs., 170 lbs. 3 oz. 2 dwts. 18 grs.
(269) 3 m. 5 fur. 39 p. 1 yd. 1 ft. 8 in.
(270) 712 fur. 10 p. 1 yd. 2 ft.
(271) 186 m. 20 p. 4 yds. 2 ft. (272) 1318 fur. 11 p. 1 ft. 6 in.
(273) 91 m. 3 fur. 15 p. 5 yds. 1 ft. 3 in.
(274) 49 m. 2 fur. 31 p. 3 yds. 1 ft. 11 in.
(275) 37 lea. 2 m. 5 fur. 23 p. 1 yd. 1 ft. 6 in.
(276) 1558 m. 6 fur. 33 p. 3 yds. 2 ft. 7 in.
(277) 124 m. 11 p. 5 yds. 3 in. (278) 439662 p. 2 ft. 7 in.
(279) 13 A. 12 P. 17 s. yds. 5 s. ft.
(280) 40 A. 1 R. 24 P. 27 s. yds. 8 s. ft.
(281) 506 R. 38 P. 27 s. yds. 4 s. ft. 99 s. in.
(282) 4 A. 3 R. 5 P. 12 s. yds. 2 s. ft. 105 s. in.
(283) 146 A. 1 R. 13 P. 9 s. yds. 4 s. ft. 109 s. in.
(284) 111 A. 2 R. 17 P. 3 s. yds. 2 s. ft. 3 s. in.
(285) 21 c. yds. 4 c. ft. 189 c. in. (286) 170 c. yds. 21 c. ft. 735 c. in.
(287) 321622 c. yds. 2 c. ft. 821 c. in.
(288) 17 qrs. 5 bus. 1 pk. 2 qts., 12 lds. 4 qrs. 7 bus. 3 pks.
(289) 48 wks. 3 d. 13 hrs. 8 min. 55 sec.
(290) 419165 m. 1 wk. 5 d. 4 hrs. 17 min.
(291) 64 tons 6 cwt. 3 qrs. 14 lbs. 10 oz. **(292) 2520.**
(293) 125 lbs. 3 oz. 5 dwts. 12 grs.
(294) 92 m. 4 fur. 18 p. 4 yds. 2 ft. 10 in.
(295) 4 wks. 4 d. 1 hr. 50 min. 43 sec.
(296) 137 m. 3 fur. 26 p. 3 yds. 1 ft. 6 in.
(297) 25. (298) 12 m. (299) 7 A. 26 P. 10½ yds.
(300) 3 c. yds. 20 c. ft. 1455 c. in. (301) 46 lbs. 3 oz. 2 dwts. 2 grs.
(302) 30 lbs. 3 oz. 1 dwt. 19 grs. (303) 192 cwt. 1 qr. 6 lbs. 4 oz.
(304) 1 ton 7 cwt. 1 qr. 23 lbs. 7 oz. (305) 3 m. 7 fur. 9 p. 2 yds. 1 ft. 5 in.

(308) 29 m. 1 fur. 1 p. 2 yds. 2 ft. 4 in.
(309) 14 c. yds. 2 c. ft. 1278 c. in. (310) 17526. (311) 50.
(312) 150561. (313) 37 lbs. 6 oz.
(314) 540. (315) 107224$\frac{8}{13}$. (316) 8 tons 1 cwt. 3 qrs. 8 lbs.
(317) 791083175. (318) 1000. (319) 2112. (320) 22862.

COMPOUND ADDITION.

(321) £8292 2*s.* 3$\frac{3}{4}$*d.* (322) £9891 15*s.* 11*d.*
(323) £10074 8*s.* 7$\frac{1}{2}$*d.* (324) £3720 14*s.* 6*d.*
(325) £42637 14*s.* 5*d.* (326) £8337 14*s.* 6$\frac{1}{2}$*d.*
(327) £4332 3*s.* 2*d.* (328) 56 cwt. 3 qrs. 10 lbs 14 oz. 13 drs.
(329) 1050 tons 4 cwt. 3 qrs. 18 lbs. 7 oz. 8 drs.
(330) 724 tons 17 cwt. 2 qrs. 24 lbs. 9 oz. 6 drs.
(331) 57 lbs. 6 oz. 12 dwts. 23 grs.
(332) 43 lbs. 5 oz. 14 dwts. 6 grs.
(333) 80 p. 3 yds. 2 ft. 11 in. (334) 195 m. 7 fur. 28 p. 3 yds. 6 in.
(335) 500 m. (336) 127 p. 12 s. yds. 3 s. ft. 8 s. in.
(337) 565 a. 24 p. 25 s. yds. (338) 124 r. 6 p. 14 s. yds. 4 s. ft. 110 s. in.
(339) 186 qrs. 2 bus. 1 gal. 3 qts. (340) 130 loads.
(α) £4506405 3*s.* 6*d.* (β) £5630662 18*s.* 5*d.*
(γ) £3493471 3*s.* 1*d.* (δ) £4371039 7*s.* 3*d.*
(ε) £3934697 13*s.* 9*d.* (ζ) £2876217 5*s.* 2*d.*

COMPOUND SUBTRACTION.

(341) £418 8*s.* 11$\frac{1}{2}$*d.* (342) £290 14*s.* 8$\frac{3}{4}$*d.*
(343) £88 12*s.* 0$\frac{1}{2}$*d.* (344) £524139 16*s.* 10$\frac{3}{4}$*d.*
(345) £2731 17*s.* 7*d.* (346) 44 tons 11 cwt. 1 qr. 23 lbs.
(347) 4 tons 18 cwt. 2 qrs. 14 lbs.
(348) 7 cwt. 3 qrs. 17 lbs. 13 oz. 15 drs.
(349) 21 lbs. 6 oz. 13 dwts. 3 grs.
(350) 1 lb. 9 oz. 2 dwts. 11 grs.
(351) 85 lbs. 10 oz. 18 dwts. 21 grs.
(352) 1 m. 5 fur. 36 p. 4$\frac{1}{2}$ yds. 1 ft. 6 in.
(353) 6 lea. 22 fur. 23 p. 4$\frac{1}{2}$ yds. 6 in.
(354) 7 a. 2 r. 36 p. (355) 1 r. 28 p. 28$\frac{1}{4}$ s. yds. 5 s. ft. 108 s. in.
(356) 4 lds. 4 qrs. 7 bus. 3 pks. (357) £16477 8*s.* 11*d.*
(358) £132 2*s.* 7$\frac{1}{2}$*d.* (359) £606 19s. 3d.
(360) £497 19*s.* 8*d.*

COMPOUND MULTIPLICATION.

(361) £30018 4*s.* $9\frac{1}{4}$*d.*, £218460 6*s.* $2\frac{1}{4}$*d.*
(362) £75834 19*s.* 8*d.*, £17902 15*s.* 3*d.*
(363) £14028 12*s.* 9*d.*, £136853 7*s.* $1\frac{1}{2}$*d.*
(364) £25799 19*s.* $5\frac{1}{2}$*d.*, £2577077 15*s.* 10*d.*
(365) £16220 12*s.* $1\frac{1}{2}$*d.*, £58415 11*s.* 10*d.*
(366) £27279 14*s.* $7\frac{1}{2}$*d.*, £366786 0*s.* 2*d.*
(367) £23337 17*s.* $1\frac{1}{2}$*d.*, £710150 12*s.* $9\frac{3}{4}$*d.*
(368) £224322 7*s.* 4*d.*, £159417 13*s.* $7\frac{3}{4}$*d.*
(369) £930 8*s.* $11\frac{1}{2}$*d.*, £6526 8*s.* $6\frac{3}{4}$*d.*
(370) £2375 5*s.* $11\frac{1}{4}$*d.*, £4179 5*s.* 10*d.*
(371) £2696 6*s.* $7\frac{1}{2}$*d*, £1973241 2*s.* $0\frac{3}{4}$*d.*
(372) 293 tons 10 cwt. 88 lbs. 4 oz.
(373) 201 tons 17 cwt. 3 qrs. 16 lbs.
(374) 611 cwt. 3 lbs. 15 drs. (375) 2073 lbs. 1 oz. 6 dwts. 21 grs.
(376) 805 fur. 12 p. 1 yd. 8 in. (377) 4334 m. 4 fur. 37 p. 4 yds.
(378) 10133 A. 2 R. 2 P. (379) 74 lbs. 3 oz. 4 dwt. 4 grs.
(380) £41 3*s.* 2*d.*

COMPOUND DIVISION.

(381) £399 7*s.* 11*d.*, £223506 6*s.* $10\frac{1}{2}$*d.*
(382) £60653 16*s.* $1\frac{1}{2}$*d.*, 2*s.* 6*d.* (383) £7 8*s.* $7\frac{1}{2}$*d.*, £23005 2*s.* 1*d.*
(384) £412 0*s.* $9\frac{3}{4}$*d.*, £26389 4*s.* $9\frac{52}{133}$*d.*
(385) 6 cwt. 3 qrs. 7 drs., 1 ft. $5\frac{11}{12}$ in. (386) 175.
(387) £299 5*s.* (388) 45 m. 7 fur. 25 p. 2 yds.
(389) £556734 10*s.* (390) 4 yrs. 45 d. 15 hrs.

---

(391) £1406 8*s.* $5\frac{2}{5}$*d.* (392) £9 18*s.* $8\frac{13}{14}$*d.*
(393) £516 10*s.* $8\frac{8}{15}$*d.* (394) £175 6*s.* $1\frac{5}{72}$*d.*
(395) £87 14*s.* $8\frac{1}{5}$*d.* (396) £794 2*s.* $2\frac{29}{40}$*d.*
(397) £190 9*s.* $10\frac{41}{48}$*d.* (398) £778 2*s.* $9\frac{11}{25}$*d.* (399) $1\frac{1}{7}$. (400) $1\frac{5}{9}$.

REDUCTION OF FRACTIONS.

(401) $10\frac{1}{2}$*d.*, 16*s.* 8*d.*, 2*s.* 1*d.* (402) £16 17*s.* 6*d.*, 13*s.* 6*d.*, £5 19*s.*
(403) £5 5*s.* 9*d.*, 8*s.* 9*d.*, 2*s.* 11*d.*

(406) £3 12*s.* 2*d.* (407) 17*s.* 8*d.* (408) £1 2*s.* $4\frac{1}{2}d.$
(409) 12*s.* $9\frac{9}{10}d.$ (410) £4 2*s.* $1\frac{33}{35}d.$ (411) £2 18*s.* $8\frac{1}{2}d.$
(412) £8 10*s.* $8\frac{17}{18}d.$ (413) 9*s.* 2*d.* (414) 2*s.* $5\frac{4}{5}d.$
(415) 14*s.* ·6*d.*
(416) 14 cwt. 1 qr. 4 lbs.; 3 oz. 8 dwts. $13\frac{5}{7}$ grs.; 3 tons 15 cwt. 1 qr. 22 lbs.
(417) 14 cwt. 7 lbs. 4 oz. 11 drs.; 61 st. 6 lbs.
(418) 10 P. 4 yds. 2 ft.; 39 A. 1 R. $1\frac{1}{4}$ P. (419) 2 qr. 17 lbs. $1\frac{7}{30}$ oz.
(420) 14 tons 3 cwt. 2 qrs. 10 lbs. 5 oz. 1 dr.
(421) 4 fur. 39 p. 2 yds. (422) 1 A. 1 R. 12 P.

---

(423) $\frac{1}{8}$, $1\frac{25}{36}$. (424) $\frac{59}{120}$, $\frac{63}{160}$. (425) $\frac{9}{10}$, $\frac{1}{3}$. (426) $\frac{349}{986}$, $\frac{5}{6}$.
(427) $3\frac{1}{7}$, $\frac{20}{21}$. (428) $\frac{7}{800}$, $\frac{299}{549}$. (429) $\frac{677}{1781}$. (430) $2\frac{14}{25}$.
(431) $1\frac{23}{224}$. (432) $\frac{2}{75}$. (433) $\frac{7}{193}$. (434) $\frac{38}{183}$.
(435) $\frac{1}{336}$. (436) $\frac{64}{605}$. (437) $10\frac{1}{25}$. (438) $\frac{83}{84}$.
(439) $\frac{99}{160}$. (440) $\frac{16}{27}$. (441) $\frac{9}{91}$. (442) $\frac{11}{28}$.
(443) $\frac{2}{7}$. (444) $\frac{1}{2}$. (445) $\frac{301}{5200}$. (446) $1\frac{2681}{3760}$.
(447) $5\frac{155}{293}$. (448) $\frac{559}{1350}$. (449) $1\frac{13}{32}$. (450) $\frac{145}{432}$.

REDUCTION OF DECIMALS.

(451) 15*s.*, 3*s.* $4\frac{1}{2}d.$ (452) 7*s.* $10\frac{1}{2}d.$, £2 17*s.* $6\frac{3}{4}d.$
(453) 15*s.* $7\frac{1}{2}d.$, £7 6*s.* $10\frac{4}{5}d.$ (454) £2 1*s.* 8*d.*, £26 3*s.* $8\frac{1}{4}d.$
(455) £2 1*s.* 9*d.*, £4 5*s.* (456) £2 12*s.* 6*d.*, 5*s.* $9\frac{3}{25}d.$
(457) £42 3*s.* $3\frac{3}{4}d.$, £3 3*s.* 7*d.* (458) 7*s.* $0\frac{1}{2}d.$, £15 16*s.* $6\frac{9}{10}d.$
(459) £121 13*s.* $11\frac{113}{125}d.$, 8*s.* $4\frac{1361}{2000}d.$
(460) £101 16*s.* $1\frac{29}{32}d.$, £5 3*s.* $6\frac{3}{10}d.$ (461) $\frac{43497}{50000}d.$, 2 tons 7 cwt. 2 qrs.
(462) 8·14968*d.*, 3 qrs. 10 lbs. 14·3616 oz.
(463) 10 tons 15 cwt. 3 qrs. $16\frac{4}{5}$ lbs., 3 cwt. 2 qrs. 14 lbs.
(464) 14 cwt. $12\frac{81}{100}$ lbs., 1 ton 18 cwt. $1\frac{21}{25}$ qrs.
(465) 5 fur. 24 p., 78 fur. 20 p.
(466) 33 A. 1 R. 32 P., 28 wks. 4 d. 9 hrs. 45 m.
(467) 7*s.* $0\frac{4}{5}d.$, 1*s.* $3\frac{3}{4}d.$ (468) 7*s.* 8*d.*, £2 2*s.* 6*d.*
(469) £33 5*s.*, 3 fur. 10 p. 3 yds. 2 ft.
(470) £2 8*s.* 11*d.*, £3 9*s.* 6*d.* (471) £1 1*s.* $0\frac{3}{4}d.$
(472) 9*s.* $3\frac{909}{1250}d.$ (473) £2 16*s.* $5\frac{1}{4}d.$
(474) 15*s.* $4\frac{7}{20}d.$ (475) £3 11*s.* 2·38125.
(476) 3 cwt. 2 qrs. $25\frac{9}{10}$ lbs. (477) 2 ft. $3\frac{609}{1000}$ in. (478) 13*s.* 9*d.*
(479) £1 9*s.* $7\frac{1159}{2500}d.$ (480) £17 6*s.* $9\frac{3}{200}d.$

---

(481) 8.625, ·0919, &c. (482) 1·71875, ·46875
(483) $\cdot 00875$, $\cdot 77291\dot{6}$. (484) $\cdot 4255\dot{9}5238\dot{0}$, $\cdot 089\dot{2}8571\dot{4}$.

(485) ·365625, ·07875. (486) ·03125, ·68$\dot{3}$.
(487) 3·625, ·$\dot{3}$. (488) 59·69565217, &c.
(489) 18·9$\dot{8}$7179$\dot{4}$. (490) ·7375. (491) ·525.
(492) ·000347$\dot{2}$, 1·68. (493) ·25, ·325.
(494) ·04910$\dot{7}$1428$\dot{5}$, 16·5. (495) ·0125, ·08125. (496) ·5.
(497) ·2$\dot{8}$1$\dot{4}$. (498) ·382$\dot{1}$4285$\dot{7}$.
(499) ·09765625. (500) £6 11*s*., ·3275.

PRACTICE.

(501) £510 10*s*., £2796. (502) £345, £7652 14*s*.
(503) £10730 10*s*., £8178 16*s*. (504) £50247 6*s*. 8*d*., £39845 8
(505) £734614 2*s*. $8\frac{1}{2}$*d*., £1756824 11*s*. 3*d*. (506) £625 19*s*. $0\frac{3}{8}$*d*.
(507) £17 2*s*. $2\frac{1}{4}$*d*. (508) £57 5*s*. $10\frac{5}{8}$*d*.
(509) £53 8*s*. 11*d*. (510) £29 11*s*. $6\frac{2}{5}$*d*.
(511) £728 17*s*. 6*d*. (512) £839 14*s*. $4\frac{1}{2}$*d*.
(513) £24 17*s*. $0\frac{9}{224}$*d*. (514) £3 9*s*. $9\frac{1}{8}$*d*.
(515) £90 0*s*. $9\frac{75}{448}$*d*. (516) £526 14*s*. $3\frac{39}{56}$*d*.
(517) £125 17*s*. $7\frac{31}{224}$*d*. (518) £1846 3*s*. 4*d*.
(519) £241 10*s*. (520) £227 14*s*.
(521) £311 8*s*. (522) £49 14*s*. $1\frac{1}{2}$*d*.
(523) £2025 9*s*. $11\frac{1}{2}$*d*. (524) £46 16*s*. $1\frac{11}{28}$*d*.
(525) £1381 15*s*. $0\frac{3}{4}$*d*. (526) £4 9*s*. $5\frac{37}{48}$*d*.
(527) £20 13*s*. (528) £36 10*s*. $2\frac{1}{4}$*d*.
(529) £269 19*s*. $8\frac{1}{4}$*d*. (530) £90 0*s*. $2\frac{457}{1440}$*d*.
(531) £2 15*s*. $11\frac{1}{4}$*d*. (532) £39921 17*s*. 6*d*.
(533) £343 12*s*. $4\frac{3}{4}$*d*. (534) £63 12*s*. $3\frac{9}{16}$*d*.
(535) £281 12*s*. $4\frac{1}{18}$*d*. (536) £669 2*s*. $9\frac{3}{4}$*d*.
(537) £417 3*s*. 9*d*. (538) £1073 11*s*. $11\frac{3}{4}$*d*.
(539) £1066 1*s*. $6\frac{3}{4}$*d*. (540) £50 2*s*. 3*d*.
(541) £54 16*s*. 6*d*. (542) £26 5*s*. $11\frac{1}{4}$*d*.
(543) £113 17*s*. $3\frac{17}{18}$*d*. (544) £1232 13*s*. $6\frac{1}{2}$*d*.
(545) £953 14*s*. 8*d*. (546) £3 10*s*. $3\frac{3}{4}$*d*.
(547) £7 12*s*. (548) £4595 8*s*. 4*d*.
(549) £414 11*s*. 3*d*. (550) £1281 19*s*. $0\frac{3}{4}$*d*.
(551) £1912 14*s*. (552) £4502 1*s*. $9\frac{1}{4}$*d*.
(553) £2210 12*s*. $3\frac{1}{2}$*d*. (554) £2759 7*s*. $8\frac{1}{2}$*d*.
(555) £17 18*s*. 11*d*. (556) £32 6*s*. 3*d*.
(557) £1727 4*s*. $6\frac{3}{4}$*d*. (558) £641 14*s*. $4\frac{1}{2}$*d*.

£216 19*s.* 5$\frac{7}{16}$*d.* (562) £223 11*s.* 3*d.*
£10330 18*s.* 2$\frac{3}{4}$*d.* (564) £8 9*s.* 10$\frac{1}{8}$*d.*
£135 14*s.* 9$\frac{3}{10}$*d.*

SINGLE RULE OF THREE.

£24 13*s.* (567) 93 bus. 2 pks. (568) 1*s.* 6*d.*
m. 1378$\frac{3}{8}$ yds. (570) 531$\frac{1}{4}$. (571) £9.
£12. (573) 5 hrs. 45 min. P.M. (574) £2 10*s.* 10$\frac{1}{2}$*d.*
s. (576) £3500. (577) 7*s.* 6*d.*
(579) 151$\frac{2}{5}$ lbs. (580) 5$\frac{7}{27}$.
7*s.* 6*d.* (582) £500. (583) £7.
$\frac{1}{2}$ m. (585) £9 6*s.* 4$\frac{1}{2}$*d.* (586) 78 A. 32 P.
0 days. (588) £8 11*s.* 10$\frac{1}{2}$*d.* (589) 24$\frac{3}{38}$.
1000. (591) £10 16*s.* (592) £124.
$\frac{21}{26}$ hrs. (594) 200 miles. (595) 2 years.
0. (597) £55 3*s.* 6$\frac{1}{4}$*d.* (598) 17*s.* 6*d.*
93 14*s.* 4$\frac{3}{4}$*d.* (600) 37 lbs. (601) £174 12*s.*
3 11*s.* 3$\frac{3}{11}$*d.* (603) £2881 16*s.* 8*d.* (604) £27 8*s.* 6$\frac{3}{4}$*d.*
33 11*s.* 3*d.* (606) £8 15*s.* (607) £2417.
25557 18*s* 1$\frac{1}{2}$*d.* (609) £3 17*s.*
5$\frac{1}{2}$. (611) 8 hrs. (612) £238.
cwt. 3 qrs. 14 lbs. (614) 4 oz. 13$\frac{1}{2}$ drs.
75 5*s.* 5$\frac{47}{248}$*d.* (616) 2*s.* 5*d.* (617) £7.
1849 11*s.* 6*d.* (619) 4677$\frac{1}{2}$. (620) £7144 7*s.* 6*d.*
0 oz. (622) £90. (623) £31 7*s.*
044 lbs. 2 oz. (625) £5. (626) 65 lbs.
90. (628) £20 1*s.* (629) £78 7*s.* 11$\frac{15}{196}$*d.*
4 13*s.* 2$\frac{1}{4}$*d.* (631) 4*s.* (632) £5583 6*s.* 8*d.*
10 19*s.* (634) 6$\frac{3}{4}$ hrs. (635) 12800.

DOUBLE RULE OF THREE.

0. (637) 27. (638) 10 days. (639) 16 cwt.
£8 16*s.* 3*d.* (641) 312. (642) 2*s.* 1*d.*
4$\frac{1}{4}$ m. (644) 40*s.* 10*d.* (645) £100. (646) 45 days.
6$\frac{2}{5}$. (648) 6440. (649) 22$\frac{6}{7}$. (650) 2340 lbs.
£57 12*s.* (652) 32 days. (653) £2 8*s.* 2*d.* (654) 12 hrs.
£50 10*s.* (656) 14400. (657) 400.
326 13*s.* 4*d.* (659) 8. (660) 45.
7 days. (662) 88 A. 2 R. 15 P. (663) 27.

(664) 240. (665) $14\frac{2}{3}$ wks. (666) 35. (667) 6664.
(668) 33 st. 13 lbs. 7 oz. $1\frac{7}{9}$ drs. (669) $11\frac{1}{25}$. (670) 21.
(671) 73. (672) 24. (673) 450. (674) $31\frac{1}{17}$.
(675) £140. (676) $17\frac{1}{9}$ days. (677) £2000. (678) $9\frac{2}{3}$.
(679) 132 days. (680) 84.

AREAS, &c.

(681) 277 s. yds. 319 s. in. (682) 104. (683) £13 9*s.* $10\frac{1}{2}$*d.*
(684) £11. (684) 136 yds. 2 ft. 8 in.
(686) 106 yds. 2 ft. (687) £2 5*s.* (688) 100.
(689) 180 yds. (690) 140 yds. (691) 208.
(692) 450. (693) 450. (694) $13\frac{1}{2}$ c. yds.
(695) $51\frac{2}{3}$. (696) 71. (697) £2 1*s.* 9*d.*
(698) 23. (699) $82\frac{2}{3}$. (700) 2 c. ft. 1457 c. in.
(701) 15 cwt. (702) 144. (703) $46\frac{7}{8}$.
(704) $\frac{1}{13}$ in. (705) $2801\frac{4}{171}$.
(706) 47 tons 17 cwt. 66 lbs. (707) 157 yds.
(708) 195 s. yds. (709) 4 s. yds. 1 s. ft. 118·4836 s. in.
(710) 1302·509. (711) $4\frac{4}{7}$ ft. (712) $567\frac{7}{9}$.
(713) 5 cwt. 2 qrs. 11 lbs. (714) 10 tons. (715) £9 19*s.* $8\frac{2}{3}$*d.*
(716) 1237500. (717) £13 4*s.* $3\frac{5}{24}$*d.* (718) $2\frac{1}{4}$ ft.
(719) £34 2*s.* (720) 654 s. ft. 72 s. in.

DUODECIMALS.

(721) 42 ft. 1′ 8″, 42 ft. 20 in.; 92 ft. 2′ 5″, 92 ft. 29 in.
(722) 273 ft. 5′ 3″, 273 ft. 63 in.; 396 ft. 5′, 369 ft. 60 in.
(723) 144 ft. 8′ 5″ 3‴, 144 ft. $101\frac{1}{4}$ in.; 42 ft. 6′ 6″ 4‴ 11⁗, 42 ft. $78\frac{59}{144}$ in.
(724) 12 ft. 9′ 0″ 0‴ 8⁗ 8′′′′′, 12 ft. $1296\frac{107}{144}$ in.
(725) 37 ft. 2′ 2″ 8‴ 1⁗ 4′′′′′, 37 ft. $320\frac{1}{9}$ in.
(726) 70 ft. 3′ 8″ 10‴ 4⁗, 70 ft. $44\frac{31}{36}$ in.
(727) 9 ft. 2′ 7″ 2‴ 5⁗, 9 ft. $31\frac{29}{144}$ in.
(728) 447 ft. 8′ 0″ 10‴ 8⁗, 447 ft. $96\frac{8}{9}$ in.
(729) 55 ft. 0′ 8″ 5‴ 2⁗, 55 ft. $8\frac{31}{72}$ in.
(730) 273 ft. 7′ 5″ 3‴, 27 s. ft. 1071 in.; 1008 c. in.

INTEREST AND DISCOUNT.

(731) £85. (732) £48 2*s.* 6*d.* (733) £277 7*s.* $6\frac{3}{10}$*d.*
(734) £687 2*s.* $10\frac{1}{5}$*d.* (735) £819 18*s.* $4\frac{39}{50}$*d.* (736) £1 4*s.* $8\frac{3}{8}$*d.*
(737) £499 19*s.* $10\frac{7}{8}$*d.* (738) £1310 15*s.* 5*d.* (739) £600 15*s.* 9·54*d.*
(740) £98 17*s.* 0·1875*d.* (741) £173 2*s.* 9*d.* (742) £200 [illegible]

£166 10*s.* $0\frac{1}{2}$·7*d.*   (744) £18496 18*s.* 4*d.*   (745) £862 15*s.* 4*d.*
£985 4*s.* 8*d.*   (747) £1246 4*s.*   (748) £1422 17*s.*

£113 9*s.* $9\frac{3}{5}$*d.*   (750) £195 2*s.*
£235 18*s.* $1\frac{3}{4}$·56*d.*   (752) £15 12*s.* $9\frac{1287}{3125}$*d.*   (753) £43 6*s.* $11\frac{1}{4}$*d.*

£1005 1*s.* $11\frac{1}{2}$*d.*   (755) £9426 5*s.*, nearly.
£880 13*s.* $7\frac{179}{400}$*d.*   (757) £267 2*s.* $5\frac{1}{4}$·09*d.*
£8508 10*s.* $10\frac{1}{2}$*d.*   (759) £5627 10*s.* $10\frac{1}{2}$*d.*, nearly.
£4854 4*s.* $0\frac{1}{2}$*d.*, nearly.

$3\frac{1}{2}$.   (762) £475.   (763) £1844 17*s.* $10\frac{466}{491}$*d.*   (764) 3 yrs.
$4\frac{1}{4}$.   (766) 4 yrs.   (767) $3\frac{1}{33}$ yrs.

£16 12*s.* $5\frac{23}{107}$*d.*   (769) £93 4*s.* 4*d.*   (770) £108 15*s.*
£31 17*s.*   (772) 6*s.* $1\frac{863}{1009}$*d.*   (773) 5*s.* 8*d.*, nearly.
£3 0*s.* 6*d.*   (775) £4 2*s.* 6*d.*

£1125.   (777) £2275.   (778) £1158 17*s.* 6*d.*

$4\frac{1}{4}$.   (780) £36 12*s.* $1\frac{1}{2}$*d.*, nearly.
£3098 6*s.* $2\frac{3}{4}$*d.*   (782) $16\frac{2}{3}$ yrs.   (783) £260 16*s.* $2\frac{31}{226}$*d.*
£668 10*s.* $6\frac{3}{4}$*d.*   (785) £11 19*s.* $9\frac{3}{121}$*d.*
5 yrs.   (787) £424 6*s.* $10\frac{2}{19}$*d.*
£14 17*s.* 6*d.*   (789) £13 17*s.* $4\frac{17}{25}$*d.*
£1604 5*s.*   (791) £893 0*s.* $1\frac{1}{2}$*d.*, nearly.
8 yrs.   (793) £9 16*s.* 8*d.*
£299 4*s.* 0·03*d.*   (795) £4484 13*s.* 9*d.*
$3\frac{3}{4}$.   (797) £48 9*s.*
£987 10*s.* $9\frac{3644}{7459}$*d.*   (799) £7600 12*s.* $2\frac{11324}{22221}$*d.*
£71 4*s.* $1\frac{1}{2}$*d.*, nearly.   (801) £70.   (802) $4\frac{3}{4}$.
£919 12*s.* 8*d.*   (804) $4\frac{1}{4}$.
Discount £3 5*s.* $9\frac{47}{77}$. Error in favour of the discounter 1*s.* $9\frac{13}{231}$*d.*
£7 15*s.* $2\frac{2}{5}$*d.*   (807) £262 4*s.* $5\frac{1}{3}$*d.*
£9 16*s.* $7\frac{1}{8}$*d.*   (809) £38 2*s.*
£214 7*s.* $1\frac{5}{11}$*d.*   (811) 4.
£21 19*s.* $1\frac{52}{89}$*d.*   (813) £42 5*s.* 10*d.*

(814) $2\frac{1}{4}$. (815) £36 13*s*. 1·44*d*.

(816) £34 17*s*. $5\frac{1}{2}$*d*. (817) £1306 5*s*.

(818) 3. (819) $4\frac{1}{2}$ yrs. (820) £173 17*s*. 6*d*.

(821) £334 5*s*. 3*d*., nearly. (822) £26 3*s*. $1\frac{3}{4}$*d*., nearly.

(823) £1129 3*s*. 4*d*. (824) $4\frac{1}{3}$. (825) $4\frac{1}{2}$ yrs.

(826) £75 6*s*. 8*d*. (827) £5856 11*s*. 5*d*.

(828) 5. (829) £2 15*s*. 3*d*. (830) 4.

COMMISSION, INSURANCE, &c.

(831) 10 gui. (832) £3 13*s*. 9*d*. (833) £21.

(834) 27*s*. (835) £4 13*s*. $11\frac{1}{20}$*d*. (836) £4000.

(837) £10 16*s*. (838) 19*s*. 6*d*. (839) £136 13*s*. 4*d*.

(840) £8820. (841) £25 10*s*. $7\frac{1}{2}$*d*. (842) £1500.

(843) £2107 10*s*. (844) £30. (845) £74 2*s*.

STOCKS.

(846) £2100. (847) £1980. (848) £1270.

(849) £9178 6*s*. 3*d*. (850) £1531 13*s*. $2\frac{7}{10}$*d*. (851) £588 5*s*.

(852) £2400, £42. (853) £4400. (854) £21 5*s*.

(855) £304 6*s*. $11\frac{11}{23}$*d*. (856) £5 13*s*. $1\frac{601}{707}$*d*.

(857) £58 10*s*. increase. (858) £55 2*s*. $8\frac{16}{37}$*d*.

(859) £19000. (860) £800. (861) £$15\frac{15}{721}$.

(862) £27. (863) £527 2*s*. $4\frac{28}{59}$*d*. (864) $3\frac{113}{229}$.

(865) £8 8*s*. $9\frac{3}{5}$*d*. increase. (866) £1600, 4 per cent.

(867) 101. (868) £29 16*s*. $7\frac{63}{263}$*d*. increase.

(869) £52 10*s*. increase. (870) £248 5*s*. 4*d*.

(871) $91\frac{1}{4}$. (872) $88\frac{6}{13}$. (873) $128\frac{10}{13}$.

(874) $18\frac{74}{377}$. (875) £7092 3*s*. $11\frac{21}{47}$*d*., $8\frac{24}{47}$.

(876) $82\frac{1}{7}$. (877) £181 1*s*. $5\frac{601}{2407}$*d*.

(878) £115 4*s*. increase. (879) £$120\frac{40}{51}$.

(880) £10020. (881) £75. (882) $3\frac{111}{763}$, $4\frac{26}{31}$.

(883) 25 per cent. increase. (884) Lost £42. (885) £9060.

PROFIT AND LOSS.

(886) 20. (887) $11\frac{1}{2}$*d*. (888) 5. (889) 16*s*. 8*d*.

(890) 18*s*. 4*d*. (891) £1 4*s*. $5\frac{33}{155}$*d*. (892) 40.

(893) $96\frac{4}{11}$. (894) 11*s*. $7\frac{23}{43}$*d*. (895) 19*s*. 2*d*.

(896) Gained 5 per cent. (897) 5*s*. 6*d*. (898) $37\frac{1}{2}$.

(902) 15. (903) £13 7*s.* 9*d.* (904) £7 9*s.* 2*d.*
(905) £2 16*s.* 3*d.* (906) 4 per cent.
(907) $5\frac{3}{5}$. (908) 10*s.* 6*d.*, $8\frac{24}{127}$.
(909) £1 1*s.* $6\frac{1}{16}$*d.*, £7 16*s.* $2\frac{286}{461}$*d.* per cent. (910) 12*s.* 6*d.*
(911) Loss £1 7*s.* $7\frac{1}{2}$*d.*, or $6\frac{63}{137}$*d.* per cent. (912) 50.
(913) 18*s.* 9*d.* per gal. (914) Gains $5\frac{85}{123}$*d.* per cent.
(915) £274 13*s.* 9*d.* (916) £24 2*s.* $3\frac{11}{47}$*d.*
(917) £1 0*s.* $9\frac{25}{26}$*d.*

PROPORTIONAL PARTS.

(918) £35049 12*s.*, £81782 8*s.*, £105148 16*s.*, £128515 4*s.*
(919) 2550, 1700, 1275, 1020, 850.
(920) £10 13*s.* 4*d.*, £13 6*s.* 8*d.*, £16.
(921) £1250, £1406 5*s.*, £1562 10*s.*, £781 5*s.* (922) 9, 42, 57.
(923) £40, £30, £42. (924) £680, £816, £714, £935.
(925) £24 10*s.*, £24, £20.
(926) £179 8*s.* 8*d.*, £142 9*s.*, £99 3*s.* 4*d.*
(927) £795, £840. (928) £750, £500, £250
(929) 12*s.* 6*d.*, 12*s.* 6*d.*, £1 5*s.*, £2 10*s.*, £5, £10.
(930) £3200, £4800, £6000, £7000.
(931) A., £59 12*s.*; B., £14 18*s.*; C., £74 10*s.*
(932) 10*s.* 6*d.* A. receives £123 7*s.* 6*d.*, B. £170 12*s.* 6*d.* C. £275 12*s.* 6*d.* (933) 2 tons 8 cwt. 2 qrs.
(934) £1024, £1280. (935) £11 19*s.* $4\frac{1}{2}$*d.*
(936) A., £190 12*s.* 6*d.*; B., £90 12*s.* 6*d.*; C., £18 15*s.*
(937) Nitre, $30\frac{2}{5}$ cwt., sulphur, $3\frac{4}{5}$ cwt., charcoal, $5\frac{4}{5}$ cwt.

EXCHANGE.

(938) £1 = 24·9 f. (939) 33*s.* 4*d.* (940) 10 gul.
(941) $10\frac{11}{16}$. (942) £1 19*s.* $4\frac{56}{127}$*d.* (943) 1468.
(944) $2436\frac{17697}{19840}$ dol. (945) £2 3*s.* $2\frac{52774}{85007}$*d.*
(946) 377 fl. 2 st. 2·4 pen. (947) $1079\frac{2}{5}$ lir. (948) $14\frac{1}{6}$.

EQUATION OF PAYMENTS.

(949) 17 mo. (950) 3 mo. (951) $3\frac{1}{2}$ mo. (952) $4\frac{5}{8}$ mo.
(953) 21 mo. (954) 12 mo. (955) $7\frac{1}{2}$ mo.

SQUARE ROOT.

(956) 19, 55. (957) 94, 111. (958) 223, 623.
(959) 711, 799. (960) 922, 977. (961) 989, 1111.

(962) 362, 53·43. (963) 327, 729. (964) 7047, 19·49.
(965) 6·005, ·091. (966) 9·00009, 34·5761.
(967) ·23046, 2·2222. (968) ·00135, 72639459.
(969) 3·16227, 2·68022. (970) 3·60554, 2·64635.
(971) $5\frac{2}{3}$, $50\frac{3}{19}$.

CUBE ROOT.

(972) 31, 38. (973) 15, 93. (974) 103, 159·85, &c.
(975) 837, 982. (976) 1061, ·623. (977) 512, 22·3.
(978) 9·77, 10·02. (979) ·03859, &c., ·074. (980) 1·259, 2·924.
(981) 1·794, ·368.

AVERAGES AND PERCENTAGES.

(982) 35·410$\dot{7}$1428$\dot{5}$. (983) £272 6*s.* $1\frac{11}{13}$*d.* (984) 26·5435$\dot{4}$1$\dot{6}$.
(985) 4·$\dot{6}$44351$\dot{4}$. (986) 22·125. (987) 26·1509125.
(988) 10804070, &c. (989) £297 15*s.* 3*d.* (990) 18495000.
(991) $13\frac{1}{3}$. (992) £2700. (993) 120000.
(994) 3·129, &c. (995) 55·83872 lbs., 48·4568 lbs., 7·70448 lbs.
(996) Decrease ·03283, &c. (997) £91 1*s.*
(998) 24813·23. (999) 1541. (1000) 11·18, nearly.
(1001) £5000, £7000

(1002) 7 fl. 8c. $7\frac{1}{2}$ m. (1003) 13 yds., 169 s. yds.
(1004) £22 18*s.* 4*d.* (1005) 30 min. (1006) 4 fl. 4 c. $2\frac{1}{4}$ m. nearly.
(1007) £267 9 fl. $9\frac{3}{8}$ m. (1008) 576. (1009) 65·041$\dot{6}$.
(1010) 7908·842$\dot{9}$7. (1011) 5012 l. 2 fl. 3 c. $7\frac{1}{2}$ m.
(1012) 17 c. ft. 415 c. in. (1013) 3*s.* 4*d.*
(1014) 3 min. 46·8 sec. (1015) $18\frac{186}{811}$. (1016) 7.
(1017) 2*s.* $11\frac{5}{9}$*d.* (1018) 765·102. (1019) $26\frac{5}{24}$.
(1020) 3 : 13. (1021) 14 l. 2 fl. 7 c. 5 m., nearly.
(1022) 220. (1023) 54 gals. (1024) 640·9 days.
(1025) 14·282 lbs. (1026) 50 days, $21\frac{3}{7}$ days.
(1027) 1 yd. (1028) $10\frac{10}{11}$ min. past 2.
(1029) 24 miles. (1030) $9\frac{4}{5}$. (1031) $1\frac{11}{28}$ days.
(1032) 1609·306. (1033) 32·19 ft. (1034) 416 yds.
(1035) $7\frac{1}{2}$ per cent., 3 dwts. $8\frac{19}{40}$ grs.
(1036) A. £4000, B. £3000, C. £2400, D. £2600.
(1037) 4 per cent (1038) 40 years.

8 hrs. $10\frac{10}{11}$ min., 8 hrs. $27\frac{3}{11}$ min., 8 hrs. $43\frac{7}{11}$ min.
$45\frac{1}{2}$, $10\frac{16}{53}$. (1041) $23\frac{41}{408}$ lds. (1042) 6 hrs.
£1203 15*s*. (1044) In 120 days, 16 min. past 12 o'clock and 14 min. before 12 o'clock. (1045) 3 hrs. 0 min. $9\frac{3}{5}$ sec.
$2\frac{22}{31}$*d*., $2\frac{1}{31}$*d*., $1\frac{18}{31}$*d*. (1047) 8 : 15.
10 hrs. 55 min. (1049) $\frac{1}{7}$. (1050) £18630.
6*s*. $8\frac{1}{2}$*d*., nearly. (1052) 211140.
$4\frac{1}{2}$ miles an hour. (1054) £27691 13*s*. 4*d*., 5 per cent.
$2\frac{1}{2}$ years after borrowing the first. (1056) 34 miles.

APPENDIX.

THE METRICAL SYSTEM.

1. An Act of Parliament legalising the use of the metrical system in Great Britain having been passed during the last session, it is necessary to give a fuller account of it than that in Art. **84.**

As there stated, the multiples and submultiples of the different units are on the decimal scale. The former are denoted by prefixes derived from the Greek, and the latter by corresponding ones derived from the Latin. Thus:

Deca (δέκα)	signifies	10 times;	deci (decem)	the	10th part
Hecto (ἑκᾰτὸν)	„	100 „	centi (centum)	„	100th „
Kilo (χί̄λιοι)	„	1000 „	milli (mille)	„	1000th „
Myria (μῡ́ριοι)	„	10000 „			

Obs. The prefixes which express division all end in *i*, the others in *a* or *o*.

2. The *Mètre* (Greek *μέτρον*, a measure) is the unit of linear measure, and from it all the others are derived. Its length is equal to the ten-millionth part of a quadrant of the meridian, or the distance between the equator and the pole. The length of the quadrant was deduced from the measurement of an arc of the meridian between Dunkirk and Barcelona, and found to be 32808992 English feet. The mètre, therefore, =39·37079 inches. Subsequent researches, however, have shown that the true length of a quadrant of the meridian is 32813000 feet, so that the mètre is ·0004008 ft. shorter than it professes to be.

The Are is the unit of surface or square measure. It is a square decamètre (10 mètres), and=119·6033 sq. yards, or 1076 sq. feet nearly.

The unit usually employed in measuring land is the *Hectare* (100 ares), and=2 A. 1 R. 35 P. nearly.

The *Stère* is the unit of solid or cubic measure. It is the cube of a mètre, and=35·3166 c. feet.

The *Litre* is the unit of the measure of capacity. Its content is equal to the cube of a decimètre, and=61·0271 c. inches.

The *Gramme* is the unit of weight. It is the weight of a cubic centimètre of distilled water at the temperature of 0° C. (32° Fahr.), and=15·4323 grs. troy.

METRICAL MEASURES AND WEIGHTS WITH ENGLISH EQUIVALENTS.

3.

Linear Measure.

	in.	ft.	yd.
1 Millimètre =	·03937079.		
1 Centimètre =	·39370790.		
1 Decimètre =	3·93707900.		
1 Mètre =	39·37079000 =	3·28089916 =	1·09363305.
1 Decamètre =	393·70790000 =	32·80899166 =	10·93633055.
1 Hectomètre =	3937·07900000 =	328·08991666 =	109·36330555.
1 Kilomètre =	39370·79000000 =	3280·89916666 =	1093·63305555.

The *Kilomètre* is the metrical mile=5 furlongs nearly.

Square Measure.

	sq. yds.	
1 Centiare =	1·196033.	
1 Are =	119·603326.	
1 Hectare =	11960·332590 =	2 A. 1 R. 35 P. nearly.

CUBIC MEASURE.

		c. ft.	c. yds.
1 Decistère	=	3·531658.	
1 Stère	=	35·316581 =	1·308021.
1 Decastère	=	353·165807 =	13·080215.

MEASURE OF CAPACITY.

		c. in.	
1 Millilitre	=	·061027 =	·0070432 gills.
1 Centilitre	=	·610271 =	·0704310 „
1 Decilitre	=	6·102705 =	·7043094 „
1 Litre	=	61·027052 =	1·7607734 pts.
1 Decalitre	=	610·270520 =	8·8038671 qts. = 2·2009668 gals.
1 Hectolitre	=	6102·705200 =	22·0096677 gals.
1 Kilolitre	=	61027·052000 =	220·0966767 gals. = 27½ bus. nearly.

WEIGHTS.

1 Milligramme	=	·015432	grs. troy	
1 Centigramme	=	·154324	„	
1 Decigramme	=	1·543235	„	
1 Gramme	=	15·432349	„	
1 Decagramme	=	154·323488	„	= 5·643830 drs. avoir.
1 Hectogramme	=	1543·234880	„	= 3 oz. 8·438302 drs. avoir.
1 Kilogramme	=	15432·348800	„	= 2 lbs. 3 oz. 4·3831 „
1 Myriagramme	=	154323·488000	„	= 22 lbs. 0 oz. 11·8305 „
1 Quintal (100 kilos)	=	1543234·88 grs. troy = 1 cwt. 3 qrs. 24 lbs. 7 oz. 6·3027 drs. avoir.		
1 Millier or Marine ton (1000 kilos)	=	15432348·8 grs. troy = 19 cwt. 2 qrs. 20 lbs. 9 oz. 15·0372 drs. avoir.		

---

1 Yard = ·91438 mètre
1 Acre = ·40467 hectare
1 Gallon = 4·54346 litres
1 Grain (troy) = ·06479 gramme
1 lb. (avoir.) = ·45359 kilogramme
1 cwt. = 50·80238 kilogrammes.

4. The metrical system was introduced and its use made compulsory in France in the year 1795, during the French

revolution; but the common people showed such an irreconcileable aversion to the decimal system, that the *Système Usuel*, or Binary System, was introduced in 1812 for the accommodation of retail trade. It has the metrical standards for its basis; but their divisions, instead of being decimal, are chiefly binary, i. e. the multiples and submultiples are obtained by doubling and halving the respective units. The decimal system was, however, retained in all government transactions, in wholesale trade, and in scientific enquiries, to which it is certainly much better adapted than our own.

5. The advantages which would arise from the adoption of a uniform system of measures and weights in all countries having commercial transactions with each other must be obvious to everyone; but it is very questionable whether they would not be more than counterbalanced by the confusion into which retail trade and the every-day affairs of life would be thrown, and the inconvenience occasioned, especially amongst the poorer classes.

MONEY.

6. In France the unit of account is the *franc*, which is equivalent to about 9½*d.* sterling. It is divided into 100 centimes. The coins are as follows:—

Gold pieces of 40 francs worth 31*s.* 8¾*d.* sterling, and of 20 francs, called *Napoleons*, worth 15*s.* 10½*d.* sterling.

Silver pieces of 5, 2, 1, ¾, ½, and ¼ francs.

Copper pieces of 20, 10, 5, 3, 2, and 1 centimes.

PATERNOSTER ROW, LONDON: *April* 1864.

GENERAL LISTS
OF
SCHOOL-BOOKS
PUBLISHED BY
MESSRS. LONGMAN, GREEN, AND CO.

◆

English Reading-Lesson Books.

Stevens and Hole's Grade Lesson-Book Primer	*Nearly ready.*
— — — Grade Lesson Books, in Six Parts	5s. 3d.

The First Standard	6d.	The Fourth Standard	9d.
The Second Standard	9d.	The Fifth Standard	1s. 3d.
The Third Standard	9d.	The Sixth Standard	1s. 3d.

Answers to the Arithmetical Exercises in Standards II. III. and IV. 4d. in Standard V. 4d. and in Standard VI. 3d. or complete, price 1s.

M'Leod's First Reading-Book, for Families and Schools	3d.
— Reading Lessons, on 30 large Broad-side Sheets	8s.
— Second Reading Book, for Families and Schools	8d.
— My First School-Book to teach Reading and Writing	6d.
— My Second School-Book to teach Spelling and Reading	9d.
Laurie's First Steps to Reading, adapted to Standard I. of the Revised Code, PART I. 3d.; PART II. 6d.; Complete, 10d.	
Laurie's Graduated Series of Reading-Lesson Books, 5 vols.	10s.
— First Reading-Lesson Book, adapted to Standard II. of the Revised Code	1s.
— Second Reading-Lesson Book, adapted to Standards III. and IV. of the Revised Code	1s. 6d.
— Third Reading-Lesson Book, adapted to Standard V. of the Revised Code	2s.
— Fourth Reading-Lesson Book, adapted to Standard VI. of the Revised Code	2s. 6d.
— Fifth Reading-Lesson Book, containing Suitable Elementary Reading for Special Highest Classes in large Schools	3s.
Simple Truths from Scripture, arranged as a Reading-Book	6d.
Jones's Secular Early Lesson-Book, Part I. 6d.; Part II.	4d.
— Advanced Reading-Book: Lessons in English History	10d.
Paul's Reading Book for Evening Schools	1s. 6d.
Marcet's Willy's Stories for Young Children	2s.
— Willy's Travels on the Railroad	2s. 6d.
— Seasons, or Stories for very Young Children, 4 vols.	each 2s.
Readings in English Prose Literature	3s. 6d.
Hughes's Select Specimens of English Prose	4s. 6d
Sullivan's Literary Class-Book	2s. 6d.
Mann's Lessons in General Knowledge	3s. 6d.

London: LONGMAN, GREEN, and CO. Paternoster Row.

Writing-Books.

M'Leod's Graduated Series of Nine Copy-Books .... each	3d.

1. Exercises in Right lined letters.
2. Exercises in Curved and Crotchet Letters.
3. Exercises in Looped Letters, &c.
4. Capitals, Large Hand, and Half Text.
5. Capitals, Large Hand, Half Text, and Small Hand.
6. Large Hand, Half Text, and Small Hand.
7. Half Text and Small Hand.
8. Small Hand.
9. For Exercises in Dictation and Composition.

School Poetry Books.

M'Leod's Poetical Reading Book, for Families and Schools	9d.
— Second Poetical Reading-Book	1s. 8d.
Cook's First Book of Poetry for Elementary Schools	9d.
— Poetry for Schools	2s.
Readings in Poetry, from the best English Poets	3s. 6d.
Twells's Poetry for Repetition, comprising 150 short pieces	2s. 6d.
Connon's First Four Books of Milton's Paradise Lost	3s. 6d.
Hughes's Select Specimens of English Poetry	3s. 6d.
Graham's Studies from the English Poets	7s.
Playtime with the Poets; a Selection for the use of Children, by a LADY	5s.

English Spelling-Books.

Mongan's Practical Spelling-Book, Pronouncing, Explanatory, and Derivative	1s. 6d.
Graham's English Spelling, with Rules and Exercises	1s. 6d.
Sewell's Dictation Exercises; a New English Spelling-Book	1s.
Carpenter's Scholar's Spelling Assistant	1s.
M'Leod's Improved Edition of Carpenter's Spelling Assistant	1s. 6d.
Sullivan's Spelling-Book superseded	1s. 4d.

Grammar and the English Language.

M'Leod's Explanatory English Grammar for Beginners	9d.
— English Grammatical Definitions, for Home Study	1d.
Bain's English Grammar	2s. 6d.
Mongan's Practical English Grammar	2s. 6d.
— Abridgment of English Grammar	*Nearly ready.*
Graham's Helps to English Grammar, or, Spelling and Reading made Easy	2s. 6d.
— English, or the Art of Composition Explained	5s.
— English Grammar Practice*	4s. 6d.
Sullivan's Manual of Etymology	10d.
— Attempt to Simplify English Grammar	1s.
Hiley's Child's First English Grammar	1s.
Abridgment of Hiley's English Grammar	1s. 9d.
Hiley's English Grammar and Style	3s. 6d.
Exercises adapted to Hiley's English Grammar	2s. 6d.; Key, 3s. 6d.
Hiley's Practical English Composition, Part I.	1s. 6d.; Key, 2s. 6d.
— Practical English Composition, Part II.	3s.; Key, 4s.
— Practical English Composition, Part III.	4s. 6d.
Sadler's Stepping-Stone to English Grammar	1s.
Best's Elementary Grammar for Village Schools	6d.
Kirkus's English Grammar, for the Junior Classes	1s.
Smart's Accidence of English Grammar	1s.
— Principles of English Grammar	2s. 6d.
Hunter's Text-Book of English Grammar	2s. 6d.
— Introduction to the Writing of Précis or Digests	2s.; Key, 1s.
— School Manual of Letter-Writing	1s. 6d.
Graham on English Style	6s.

Marcet's Willy's Grammar, for the Use of Boys	2s. 6d.
— Mary's Grammar, intended for the Use of Girls	3s. 6d.
— Game of Grammar, in Counters	8s.
— Conversations on Language	4s. 6d.
Morell's Essentials of English Grammar and Analysis .... cloth 9d.; sewed	8d.
— Grammar of the English Language, 2s.; or, with a series of Graduated Exercises	2s. 6d.
The Exercises separately, cloth 9d. sewed 8d.	
Lowres's Grammar of English Grammars	3s. 6d.
— Companion to English Grammar	2s. 6d.
Edwards's History of the English Language	9d.
— Practical Introduction to English Composition	1s. 6d.
Martin's First English Course, on the Analysis of Sentences	2s. 6d.
Latham on the English Language, Fifth Edition, 8vo.	18s.
Latham's Handbook of the English Language	7s. 6d.
— Elements of Comparative Philology, 8vo.	21s.
— English Grammar for Classical Schools	2s. 6d.
— Elementary English Grammar	4s. 6d.
Latham and Maberly's Smaller English Grammar	2s. 6d.
Brewer's Guide to English Composition	5s. 6d.
Clark's Student's Handbook of Comparative Grammar	7s. 6d.
Roget's Thesaurus of English Words and Phrases	10s. 6d.
Müller's Lectures on the Science of Language, First Series, Fourth Edit. 8vo.	12s.
— Second Series, preparing for publication.	

Paraphrasing, Parsing, and Analysis.

Morell's Analysis of Sentences Explained and Systematised	2s.
Lowres's System of English Parsing and Derivation	1s.
Hunter's Paraphrasing and Analysis of Sentences .... 1s. 3d.; Key,	1s. 3d.
— Shakspeare's *Henry the Eighth*	2s. 6d.
— Shakspeare's *Julius Cæsar*	2s. 6d.
— Shakspeare's *Merchant of Venice*	2s. 6d.
— Books I. and II. of Milton's *Paradise Lost* .... each Book	1s. 6d.
— Questions on *Paradise Lost*, I. and II. and on the *Merchant of Venice*,	1s.
— Milton's Comus, L'Allegro, and Il Penseroso	1s. 6d.
— Bacon's *Advancement of Learning*, Annotated summary of	2s.
— School Edition of Johnson's *Rasselas*	2s. 6d.
M'Leod's School Edition of Thomson's *Spring*	2s.
— — — *Winter*	2s.
— — — Goldsmith's *Deserted Village*	1s. 6d.
Bromby's School Edition of Wordsworth's Excursion, Book I.	1s. 6d.

Dictionaries; with Manuals of Etymology.

Latham's Dictionary of the English Language, founded on Dr. Johnson's, with numerous Emendations and Additions, in course of publication in Thirty Monthly Parts, forming Two Volumes, 4to. .... each	3s. 6d.
Black's Student's Manual of Words derived from the Greek	2s. 6d.
— Sequel to above, or Dictionary of Words derived from the Latin	5s. 6d.
Sullivan's Dictionary of Derivations	2s.
— Dictionary of the English Language	3s. 6d.
Graham's English Synonyms, Classified and Explained	6s.
English Synonyms, edited by R. Whately, D.D.	3s.
Smart's Walker's Pronouncing Dictionary, 8vo	12s.
— Walker's Pronouncing Dictionary Epitomised	6s.
Maunder's Treasury of Knowledge and Library of Reference	10s.
— Scientific and Literary Treasury	10s.

Elocution.

Smart's Practice of Elocution .... 4s.
— Historico-Shakspearian Readings .... 5s.
Rowton's Debater; or, New Theory of Public Speaking .... 6s.
Twells's Poetry for Repetition .... 2s. 6d.

Gleig's School Series.

A New Series of Elementary School-Books; each Book (in most instances) complete in itself, price 9d. Intended to comprise a complete Course of Elementary Education. Projected and edited by the Re.. G. R. Gleig, M.A. Chaplain-General to Her Majesty's Forces; assisted by eminent Teachers and Promoters of Education.—The various works which compose Gleig's School Series are all included in the body of the present Catalogue, each in the division to which it belongs.

Arithmetic.

Colenso's Arithmetic designed for the use of Schools .... 4s. 6d.
Maynard's Key to Colenso's Arithmetic for Schools .... 6s.
Colenso's Arithmetical Tables, on a Card .... 1d.
— Elementary Arithmetic .... 1s. 9d., or with Answers, 2s. 3d.
☞ Colenso's *Elementary* Arithmetic may be had in Five Parts, as follows:—
1. Text-Book .... *6d.* | 3. Examples, Pt. II. *Compound Arithmetic* .... *4d.*
2. Examples, Pt. I. *Simple Arithmetic* *4d.* | 4. Examples, Pt. III. *Fractions, Decimals, Duodecimals* *4d.*
5. Answers to the Examples, with Solutions of the more difficult Questions .... 1s.
M'Leod's Six Standards of Arithmetic, Standards I. and II. .... each 9d.
— Manual of Arithmetic; containing 1,750 Questions .... 9d.
— Solutions of Arithmetical Questions by First Principles, 8vo .... 2s. 6d.
— Mental Arithmetic, PART I. Whole Numbers; PART II. Fractions, each 1s.
— Extended Multiplication and Pence Tables .... 2d.
Solomon's Improved Arithmetical Tables .... 6d.
Johnston's Civil Service Arithmetic, containing 1,300 Questions .... 3s. 6d.
Hiley's Arithmetical Companion .... 2s.; Key, 5s.
Tate's Treatise on the First Principles of Arithmetic .... 1s. 6d.
— Companion to the First Principles of Arithmetic .... 3s. 6d.
— System of Mental Arithmetic, for the use of Teachers, 8vo .... 1s.
Tate On the New Coinage, in relation to School Arithmetics .... 9d.
Tate's Principles of the Differential and Integral Calculus .... 4s. 6d.
Hall's Treatise on the Differential and Integral Calculus .... 8s. 6d.
Galbraith and Haughton's Manual of Arithmetic .... sewed 3s., cloth 3s. 6d.
Pix's Miscellaneous Examples in Arithmetic .... 2s. 6d.
Davis's Memory Work of Arithmetic .... 4d.
— Arithmetical Examples. Part I.—containing 138,000 New Questions from Simple Addition to Compound Proportion ... cloth 8d.; Key 1s.
*** This Key serves also for Davis's *Grade Arithmetic* and *Cards*.
— — — Part II.—containing 3,143 Questions in the higher Rules and in Mensuration .... cloth 8d.; Key 1s.
*** *Examples*, Parts I. and II. together, 1s. 4d. The Two Keys together, 2s.
Davis's Grade Arithmetic. Part I. viz. Standards I. II. and III.—containing 2,545 Examples in the Four Simple Rules ...... 2d.
— — — Part II. viz. Standards IV. and V.—containing 3,065 Examples in the Four Compound Rules .. 2d.
— — — Part III. viz. Standard VI.—containing 782 Examples in Bills of Parcels, Proportion, and Practice 2d.
— — — Complete, the Three Parts in One vol. .... 6d.

Davis's Halfpenny Arithmetic Cards, for sale in Schools at a Halfpenny each Card. To be had only in Packets of Thirty-six Cards, as follows:—

1. Simple Rules complete, containing 970 Sums on 18 Cards, price 9d.
2. Simple Addition, 3 sets, each containing 120 Sums on 18 Cards, price 9d.
3. Simple Subtraction, 9 sets, each containing 160 Sums on 18 Cards, price 9d.
4. Simple Multiplication, 9 sets, each containing 340 Sums on 18 Cards, price 9d.
5. Simple Division, 9 sets, each containing 350 Sums on 18 Cards, price 9d.
6. Compound Rules complete, containing 1,257 Sums on 18 Cards, price 9d.
7. Compound Addition, 2 sets, each containing 147 Sums on 18 Cards, price 9d.
8. Compound Subtraction, 6 sets, each containing 142 Sums on 18 Cards, price 9d.
9. Compound Multiplication, 6 sets, each containing 488 Sums on 18 Cards, price 9d.
10. Compound Division, 6 sets, each containing 505 Sums on 18 Cards, price 9d.

Calder's Familiar Explanation of Arithmetic, price 4s. 6d.; or with Answers, price 5s. 6d.; the Answers separately, 1s.
Liddell's Arithmetic for Schools ....2s.; Answers, 3d.

Book-keeping.

Isbister's Book-keeping by Single and Double Entry .... 9d.
— Set of Eight Account Books, adapted to the above ....each 6d.
Hunter's Solutions of Questions in Arithmetic and Book-keeping, used in the Civil Service Examinations .... 1s. 6d.
— Examination-Questions in Book-keeping by Double Entry .... 1s.
Or with Answers, 2s. 6d.
Ruled Paper for the various Forms of Account Books required in Hunter's Examination-Questions in Book-keeping, 5 sorts ....per quire, 1s. 6d

Mensuration.

Hunter's Elements of Mensuration, 18mo............................9d.; Key, 9d.
Boucher's Mensuration, Plane and Solid, 12mo.................................. 3s.
Lund's Elements of Mensuration .... 3s. 6d.
Calder's Exercises in Mensuration: forming a Key to the above .... 3s.
Nesbit's Treatise on Practical Mensuration ....6s.; Key, 5s.

Algebra.

Colenso's Elements of Algebra, for the use of Schools, Part I.....4s. 6d.; Key, 5s.
Hunter's Examination-Questions on Colenso's Algebra, Part I. .... 2s. 6d.
Colenso's Elements of Algebra, for the use of Schools, Part II. ......6s.; Key, 5s.
— — — complete, 8vo. ....12s. 6d.; Key, 7s. 6d.
— — — for National Schools..............1s. 6d.; Key. 2s. 6d.
— Miscellaneous Examples and Equation Papers in Algebra .... 2s. 6d.
Tate's Algebra made Easy .... 2s.; Key, 3s. 6d.
Reynolds's Elementary Algebra, for Beginners ....9d.; Key, 3d.
Hall's Elements of Algebra, for Schools and Colleges .... 5s.
Thomson's Elementary Treatise on Algebra..........................5s.; Key, 4s. 6d.
Wood's Elements of Algebra, for Students of the University, by Lund, 8vo.....12s. 6d.
Lund's Companion to Wood's Algebra .... 7s. 6d.
— Key to Wood's Algebra .... 7s. 6d.
— Short and Easy Course of Algebra ....2s. 6d.; Key, 2s. 6d.
Galbraith and Haughton's Manual of Algebra, Part I. ......sewed, 2s.; cloth, 2s. 6d.
Wharton's Solutions of every class of Examples in Algebra..................... 6s. 6d.

Geometry and Trigonometry.

Potts's Editions of Euclid's Elements for Schools and Colleges.

Potts's Euclid's Elements, Books I. to VI. and parts of XI. and XII., University Edition, 8vo.................................10s.
— — — School Edition .... 4s. 6d.

Potts's Euclid's Elements, School Edition, Books I. to IV. 3s.; Books I. to III.	2s. 6d.
— — — with Notes, Questions, and Geometrical Exercises, Book I. 1s.; Books I. and II.	1s. 6d.
— Enunciations of Euclid	9d.
Tate's Practical Geometry; with 261 Woodcuts, 18mo.	1s.
— Principles of Geometry, Mensuration, &c. 12mo	3s. 6d.
Lund's Geometry as a Science, 1s. 6d. and Geometry as an Art, 2s.; together,	3s. 6d.
— Geometrical Easy Exercises, with their Solutions	3s. 6d.
Galbraith and Haughton's Manual of Euclid, Books I. to III. sewed, 2s.; cloth,	2s. 6d.
— — Books IV. to VI. sewed, 2s.; cloth,	2s. 6d.
Davis's Elements of Plane Geometry, or First Six Books of Euclid	1s.
Isbister's First Steps to Euclid	1s. 6d.
— Geometrical Copy-Books Two Copy-Books; each	6d.
— School Euclid ... 2s. 6d. separately; Book I. 1s. and Books I. and II.	1s. 6d.
— College Euclid	*In preparation.*
Tate's First Three Books of Euclid's Elements 12mo. 1s. 6d.; 18mo.	9d.
Narrien's Elements of Geometry, 8vo	10s. 6d.
Colenso's Elements of Euclid, 18mo. 4s. 6d.; or with Key,	6s. 6d.
— Geometrical Problems and Key	3s. 6d.
— Geometrical Problems, separately	1s.
— Plane Trigonometry Part I. 2s. 6d.; Key,	3s. 6d.
— — — Part II. 2s. 6d.; Key,	5s.
*** The Keys to the Two Parts of the Trigonometry, together, price 8s. 6d.	
Hunter's Elements of Plane Trigonometry 1s.; Key,	9d.
Galbraith and Haughton's Manual of Plane Trigonometry, sewed, 2s.; cloth	2s. 6d.
Hymers's Treatise on Plane and Spherical Trigonometry, 8vo	8s. 6d.
— Differential Equations and Calculus of Finite Differences, 8vo.	12s.
Hunter's Treatise on Logarithms 1s.; Key,	9d.
Jeans's Plane and Spherical Trigonometry 7s. 6d.; or in Two Parts, each	4s.
— Problems in Astronomy, &c. a Key to the above	6s.

Land Surveying, Drawing, and Practical Mathematics.

Tabor's Land Surveying and Levelling for Farmers and Schools	3s. 6d.
Nesbit's Treatise on Practical Land Surveying	12s.
Tate's Drawing-Book for Little Boys and Girls	1s. 6d.
— Drawing for Schools, after the Method of Dupuis	5s. 6d.
— Mathematics for Working Men, Part I. 8vo	2s.
Thornton's Elementary Treatise on Land Surveying and Levelling	2s. 6d.
Binns's Orthographic Projection and Isometrical Drawing	1s.
— Geometrical Drawing Part I. 4s.; Part II. 6s.; complete,	9s. 6d.
Hale's Handbook of Elementary Drawing	5s.
Cape's Course of Mathematics, 2 vols. 8vo	32s.
— Mathematical Tables of Logarithms, &c. royal 8vo	10s. 6d.
Winter's Elementary Geometrical Drawing Part I. 8s. 6d.; Part II.	6s. 6d.
Galbraith and Haughton's Manual of Mathematical Tables sewed, 3s.; cloth,	3s. 6d.
Kimber's Mathematical Course for the University of London, Part I. (Matriculation Examination) 8vo 6s. 6d.; Solutions, forming a Key,	2s. 6d.
Salmon's Treatise on Conic Sections, 8vo	12s.
Wrigley's Examples and Problems in Pure and Mixed Mathematics, 8vo.	8s. 6d.
Fowler's Solutions of Questions in Mixed Mathematics, 8vo.	3s. 6d.
Quarterly Journal of Pure and Applied Mathematics, 8vo.	5s.

Vocal Music.

Parkhurst's Stepping-Stone to Music	1s.
Turle and Taylor's Art of Singing at Sight	5s.

Tilleard's Collection of Sacred Music, for Schools, super-royal 8vo.	1s. 6d.
— Collection of Secular Music, for Schools, super-royal 8vo.	1s. 6d.

The above Two Collections may be had in one Volume, price 5s. cloth, gilt edges.

Tilleard's People's Chant-Book	9d.

Works by John Hullah, Professor of Vocal Music in King's College, and in Queen's College, London.

New Editions, '*Revised and Reconstructed in 1849,*' *of* 'WILHEM's Method of Teaching Singing,' *adapted to English Use, under the Superintendence of the Committee of Council on Education.*

The Manual, for the use of Teachers and Pupils, Parts I. and II. price 2s. 6d. each; or together in cloth	5s.
The Exercises and Figures contained in Parts I. and II. of the Manual, for the use of Pupils Books I. and II. price, each,	8d.
Large Sheets, containing the Figures in Part I. of the Manual, Nos. 1 to 8 in a Parcel price	6s.
Large Sheets, containing the Exercises in Part I. of the Manual, Nos. 9 to 40, in Four Parcels of Eight Nos. each price, per Parcel,	6s.
Large Sheets, containing the Figures in Part II. of the Manual, Nos. 41 to 52, in a Parcel price	9s.
Hullah's Rudiments of Musical Grammar	3s.
— Grammar of Musical Harmony, royal 8vo.	3s.
Exercises to Grammar of Musical Harmony	1s.
Hullah's Short Treatise on the Stave	2s.
Grammar of Counterpoint. Part I. 8vo.	*Nearly ready.*
Hullah's Infant School Songs	6d.
School Songs for 2 and 3 Voices. 2 Books, 8vo. each	6d.
Hullah's Exercises for the Cultivation of the Voice. For Soprano or Tenor. Third Edition	2s. 6d.
— Exercises for the Cultivation of the Voice. For Contralto or Bass. Third Edition	2s. 6d.

Hullah's Part Music.

Class A. In Score and in separate Voice Parts, for Soprano, Alto, Tenor, and Bass. Two Volumes of Sacred and Two of Secular Pieces. Score, 5s. each Volume, cloth; 4s. in wrapper; Voice Parts, 1s. in wrapper; 1s. 9d. cloth.

Class B. In Score for the Voices of Women and Children. One Volume of Sacred and One of Secular Pieces. 1s. each in wrapper; 1s. 9d. cloth.

Class C. In Score for the Voices of Men. One Volume of Sacred and One of Secular Pieces. 1s. each in wrapper; 1s. 9d. cloth.

Political and Historical Geography.

Hogarth's Outlines of Geography, for Families and Schools	10d.
The Stepping-Stone to Geography	1s.
Hughes's (W.) Child's First Book of Geography	9d.
— Geography of the British Empire	9d.
— Book of General Geography	9d.
Hiley's Child's First Geography	9d.
— Progressive Geography	2s.
Descriptive Geography, being Vol. V. of the 'Instructor'	2s.

Hughes's (W.) Manual of Geography, with Six Maps	7s. 6d.
Or, in Two Parts:—I. Europe, 3s. 6d.; II. Asia, Africa, America, Australasia, and Polynesia, 4s.	
Hughes's (W.) Manual of British Geography, with Four Maps	2s.
— Geography of British History, with Six Maps	8s. 6d.
Sullivan's Geography Generalised	2s.
— Introduction to Geography, Ancient, Modern, and Sacred	1s.
Bray's Geography of the British Empire	7s. 6d.
Lott and Hughes's Geography of India	*Just ready.*
Hughes's (E.) Geography for Elementary Schools	1s.
Marcet's Conversations on Land and Water	5s. 6d.
Goldsmith's Grammar of General Geography	3s. 6d.; Key, 1s.
Dowling's Introduction to Goldsmith's Geography	9d.
— Questions on the Maps in Goldsmith's Geography	9d.; Key, 9d.
Maunder's Treasury of Geography, with 7 Maps and 16 Plates	10s.
Butler's Sketch of Ancient and Modern Geography	7s. 6d.
— Sketch of Modern Geography	4s.
— Sketch of Ancient Geography	4s.
Cunningham's Abridgment of Butler's Geography	2s.
M'Leod's Geography of Palestine or the Holy Land	1s. 6d.
— Life and Travels of St. Paul	2s.
Johnston's (Keith) General Gazetteer, or Dictionary of Geography, 8vo	30s.

Physical and Mathematical Geography.

Zornlin's Recreations in Physical Geography	6s.
— Outlines of Physical Geography	10d.
Hughes's (E.) Outlines of Physical Geography, 12mo	3s. 6d.
Dick's Compendium of Mathematical Geography	5s.
Keith's Treatise on the Use of the Globes	6s. 6d.; Key, 2s. 6d.
Hughes's (W.) Manual of Mathematical Geography	4s. 6d.

School Atlases and Maps.

Butler's Atlas of Modern Geography, enlarged, royal 8vo. 10s. 6d.; or royal 4to.	10s. 6d.
— Junior Modern Atlas, comprising 12 Maps	4s. 6d.
— Atlas of Ancient Geography, enlarged, royal 8vo	12s.
— Junior Ancient Atlas, comprising two New Maps, in all 12 Maps	4s. 6d.
— General Atlas of Modern and Ancient Geography, royal 4to	22s.
— Outline Geographical Copy-Books, Ancient and Modern, each Copy-Book	4s.
M'Leod's New Middle-Class Atlas, 29 full-coloured Maps, 4to	5s.
— Oxford Middle Class *Junior Atlas* for 1864	1s. 6d.
— Oxford Middle Class *Senior Atlas* for 1864	2s. 6d.
— Hand-Atlas of General and Political Geography, half-bd., 3s.; sewed,	2s. 6d.
— Class Atlas of Physical Geography half-bound, 3s.; sewed,	2s. 6d.
Bowman's Questions on M'Leod's Class Atlas of Physical Geography	1s.
M'Leod's Wall-Map of England and Wales, No. I. *Physical*, price 6s. on a sheet, coloured; or price 9s. mounted or in a Portfolio.	
— — — England and Wales, No. II. *Political*, price 7s. 6d. on a sheet, coloured; or price 10s. 6d. mounted or in a Portfolio.	
— Wall-Map of England and Wales, No. III. *Geological*, price 16s. on a sheet, full-coloured; or price 20s. mounted or in a Portfolio.	
*** The above Three Maps together in a Portfolio, price 36s.	
— Physical Atlas of Great Britain and Ireland	7s. 6d.

M'Leod's Class Atlas of Scripture Geography, royal 8vo.	7s.
Hughes's (E.) General Atlas for Elementary Schools	1s. 6d.
— Atlas of Physical, Political, and Commercial Geography, rl. 8vo.	10s. 6d.
— School Atlas of Bible Lands	1s. 6d.
Brewer's Elementary Atlas of History and Geography, royal 8vo.	12s. 6d.

Geology and Mineralogy.

Phillips's Guide to Geology, with 4 Plates and 53 Diagrams	4s.
— Treatise on Geology, 2 vols.	7s.
Bristow's Glossary of Mineralogy, with 486 Figures on Wood	12s.

Natural History and Botany.

The Book of Animals, Class Mammalia	1s. 6d.
The Book of Birds, Class Aves	1s. 6d.
The Book of Fishes, Class Pisces	1s. 6d.
The Book of Reptiles, including Fossil Remains	1s. 6d.
The Stepping-Stone to Natural History, 1 vol.	2s. 6d.
Or in Two Parts, I. *Mammalia*, 1s. II. *Birds, Reptiles*, and *Fishes*, 1s.	
Owen's Natural History for Beginners, 1 vol.	2s.
Or in Two Parts, price 9d. each.	
Sister Mary's Tales in Natural History	1s. 6d.
Tate's Natural History of Familiar Things	1s.
Maunder's Treasury of Natural History; with 900 Woodcuts	10s.
Lindley and Moore's Treasury of Botany	*In the Press.*
The Book of Trees, Fourth Edition, with Woodcuts	2s.
Marcet's Lessons on Animals, Vegetables, and Minerals	2s.
Lessons on the Universe, Animal, Vegetable, and Mineral Kingdoms, and Human Form; being Vol. III. of the 'Instructor'	2s.
Swainson's Treatise on the Habits and Instincts of Animals	3s. 6d.

Chemistry.

Tate's Outlines of Experimental Chemistry	9d.
Piesse's Laboratory of Chemical Wonders	5s. 6d.
Marcet's Conversations on Chemistry, 2 vols.	14s.
Thomson's School Chemistry, or Practical Rudiments of the Science	6s. 6d.
Conington's Handbook of Chemical Analysis	7s. 6d.
— Tables of Qualitative Analysis	2s. 6d.
Odling's Manual of Chemistry, Descriptive and Theoretical, Part I. 8vo.	9s.

Natural Philosophy.

Arnott's Elements of Physics or Natural Philosophy, Part I. 8vo.	10s. 6d.
Readings in Science, Fifth Edition	3s. 6d.
Marcet's Conversations on Natural Philosophy	10s. 6d.
Tate's Science of Familiar Things, Vol. I. 3s. 6d.; or in 3 Parts	each 1s.
— Light and Heat, familiarly explained	9d.
— Hydrostatics, Hydraulics, and Pneumatics	9d.
— Electricity, familiarly explained and illustrated	9d.
— Magnetism, Voltaic Electricity, and Electro-Dynamics	9d.

Galbraith and Haughton's Manual of Optics ....sewed, 2s.; cloth,	2s. 6d.
— — Manual of Hydrostatics..........sewed, 2s.; cloth,	2s. 6d.
Downing's Elements of Practical Hydraulics, 8vo.	8s.

Mechanics and Mechanism.

Moseley's Illustrations of Practical Mechanics..................................	8s.
Tate's Exercises on Mechanics and Natural Philosophy..............2s.; Key,	3s. 6d.
— Principles of Mechanical Philosophy, 8vo................................	10s. 6d.
— Mechanics and the Steam-Engine	9d.
— Elements of Mechanism. Fifth Edition	3s. 6d.
Goodeve's Elements of Mechanism	6s. 6d.
Kater and Lardner's Treatise on Mechanics	3s. 6d.
Galbraith and Haughton's Manual of Mechanics ....sewed, 3s.; cloth,	3s. 6d.
Twisden's Elementary Introduction to Practical Mechanics	10s. 6d.

Fortification, &c.

Kimber's Vauban's First System of Fortification, 8vo.	5s.
— on the Construction of the Modern System, a New and Cheaper Edition preparing.	
— Field Works, as executed at Sandhurst and Addiscombe, 8vo...........	6s.
Macdougall's Theory of War	10s. 6d.
— Campaigns of Hannibal..	7s. 6d.

Engineering and Science.

Lowndes's Engineer's Handbook, Second Edition	5s.
The Artisan Club's Treatise on the Steam Engine, 4to.	42s.
Bourne's Catechism of the Steam Engine..	6s.
Fairbairn's Useful Information for Engineers, 2 vols. ....each	10s. 6d.
— Treatise on Mills and Millwork, 2 vols.	32s.
Moseley's Mechanical Principles of Engineering and Architecture, 8vo........	24s.

Popular Astronomy and Navigation.

Hall's Outlines of Astronomy, for Families and Schools.........................	10d.
The Stepping-Stone to Astronomy	1s.
Read's Popular and Mathematical Astronomy	3s. 6d.
Tate's Astronomy and the Use of the Globes, for Beginners.....................	9d.
Herschel's Treatise on Astronomy, fcp. 8vo.	3s. 6d.
— Outlines of Astronomy, Sixth Edition, 8vo.	18s.
Galbraith and Haughton's Manual of Astronomy............sewed, 2s.; cloth,	2s. 6d.
Arago's Popular Astronomy, translated by W. H. Smyth, D.C.L. and R. Grant, M.A. 2 vols. 8vo.	45s.
Saxby's Study of Steam and the Marine Engine	5s. 6d.
Young's Nautical Dictionary, Second Edition, 8vo.	18s.
Main and Brown's Marine Steam Engine, 8vo.	12s. 6d.
— on the Indicator and the Dynamometer, 8vo.................	4s. 6d.
— Questions and Examination-Papers on the Steam Engine, 8vo.	5s. 6d.
Jeans's Handbook for the Stars, royal 8vo..	3s. 6d.
— Navigation and Nautical Astronomy ....9s. or in 2 Parts, each	5s.
Boyd's Manual for Naval Cadets	12s. 6d.

Animal Physiology and the Preservation of Health.

Shield's Stepping-Stone to Animal and Vegetable Physiology	1s.
Bray's Physiology for Schools	1s.
Howard's Manual of Athletic and Gymnastic Exercises	7s. 6d.
Mann's Book of Health	9d.
Marshall's Physiology for Schools and Self-Instruction	*Nearly ready.*

Domestic Economy and General Knowledge.

Lessons on Houses, Furniture, Food, and Clothing; with Questions. Being Vol. II. of the 'Instructor'	2s.
Domestic Economy, edited by the Rev. G. R. Gleig, M.A.	9d.
Instructions for Girls in Household Matters	1s. 6d.
The Stepping-Stone to Knowledge	1s.
Second Series of the Stepping-Stone to General Knowledge	1s.

Chronology.

Book of the Calendar, the Months, and the Seasons; with Questions. Being Vol. IV. of the 'Instructor'	2s.
Conybeare's School Chronology, or Great Dates of History	1s.
Slater's School Chronology, or Sententiæ Chronologicæ	3s. 6d.
— Coloured Chart of Chronology and History	9s. 6d.
Valpy's Poetical Chronology of Ancient and English History	2s. 6d.
Nicolas's Chronology of History	3s. 6d.
Woodward's Historical and Chronological Encyclopædia	*In preparation.*

Mythology and Antiquities.

Hort's Pantheon, or Introduction to the Mythology of the Ancients	4s. 6d.
Rich's Illustrated Dictionary of Roman and Greek Antiquities	12s. 6d.
Lempriere's Classical Dictionary, abridged by Barker, 8vo	7s. 6d.
Cox's Tales from Greek Mythology	3s. 6d.
— Tales of the Gods and Heroes	5s.
— Tales of Thebes and Argos	4s. 6d.

Biography.

Readings in Biography, Tenth Edition	3s. 6d.
Gleig's Book of Biography	9d.
The Stepping-Stone to Biography	1s.
Maunder's Biographical Treasury, Twelfth Edition	10s.

British History.

Outlines of the History of England, Twenty-fourth Edition	1s.
Turner's Analysis of English and French History	2s.
The Stepping Stone to English History	1s.
Gleig's First Book of History—England	2s.
— British Colonies, or Second Book of History	9d.
— British India, or Third Book of History	9d.
Historical Questions in Gleig's School Series	9d.
School History of England, abridged from Gleig's Family History	6s.
Anthony's Footsteps to the History of England	3s.
Marcet's Conversations on the History of England	5s.

Lupton's English History, for Candidates for Examination	*Just ready.*
Farr's School and Family History of England	5s. 6d.
Longman's (W.) Lectures on the History of England, Vol. I. 8vo.	15s.
Scott's (Sir W.) History of Scotland, 2 vols.	7s.

General History.

Stafford's Compendium of Universal History	4s.
Keightley's Outlines of History	3s. 6d.
Maunder's Historical Treasury, with a new Index	10s.
Anthony's Footsteps to Modern History	5s. 6d.
— Footsteps to the History of France	3s.
Tales, Conversations, and Easy Lessons from History; with Questions. Being Vol. I. of 'The Instructor'	2s.
Elements of Ancient History; with Questions. Being Vol. VI. of 'The Instructor'	2s.
Elements of Modern History; with Questions. Being Vol. VII. of 'The Instructor'	2s.
The Stepping-Stone to French History	1s.
Crowe's Library History of France, 8vo. .. Vol. I. 14s.; Vol. II. 15s.; Vol. III.	18s.
Mangnall's Historical and Miscellaneous Questions	4s. 6d.
Corner's Questions on the History of Europe	5s.
Bell's Historical Sketches of Feudalism	5s. 6d.
Sewell's Ancient History of Egypt, Assyria, and Babylonia	6s.
Outlines of Grecian History, for Families and Schools	1s.
The Stepping-Stone to Grecian History	1s.
Browne's History of Greece, in Gleig's Series	9d.
Sewell's First History of Greece	3s. 6d.
Cox's Tale of the Great Persian War, from Herodotus	7s. 6d.
Taylor's Student's Manual of Ancient History	6s.
— Student's Manual of Modern History	6s.
Schmitz's School History of Greece, with Map and Woodcuts	7s. 6d.
Turner's Analysis of the History of Greece	2s.
Thirlwall's History of Greece, 8 vols. fcp. 28s.; or 8 vols. 8vo.	60s.
Mure's Language and Literature of Ancient Greece, 5 vols. 8vo.	69s.
Sewell's Child's First History of Rome	2s. 6d.
Outlines of Roman History, for Families and Schools	10d.
Parkhurst's Stepping-Stone to Roman History	1s.
Turner's Analysis of Roman History	2s.
Browne's History of Rome, in Gleig's Series	9d.
Merivale's Fall of the Roman Republic	7s. 6d.
— History of the Romans under the Empire, 7 vols. 8vo.	108s.

Scripture History and Religious Works.

The Stepping-Stone to Bible Knowledge	1s.
Gleig's Sacred History, epitomised in simple language, 2s. or 2 parts ...... each	9d.
Holme's Annotations on the Gospel of St. Mark	2s.
Conybeare and Howson's Life and Epistles of St. Paul, 2 vols.	12s.
Ellicott's Commentary on St. Paul's Epistles, 8vo. Galatians, Third Edition, 8s. 6d. Ephesians, Second Edition, 8s. 6d. The Pastoral Epistles, Second Edition, 10s. 6d. Philippians, Colossians, and Philemon, Second Edition, 10s. 6d. Thessalonians, Second Edition, 7s. 6d.	
Potts's Paley's Evidences and Horæ Paulinæ, 8vo.	10s. 6d.
Browne's Exposition of the Thirty-Nine Articles, 8vo.	16s.
Gorle's Examination Questions on the above	3s. 6d.
Ayre's Treasury of Bible Knowledge	*In the press.*

Riddle's Manual of the Whole Scripture History	4s.
— Outlines of Scripture History	2s. 6d.
Titcomb's Bible Studies, 8vo.	8s. 6d.
Horne's Introduction to the Holy Scriptures, 4 vols. 8vo.	73s. 6d.
— Compendious Introduction to the Study of the Bible	9s.
Cook's Acts of the Apostles, with a Commentary	5s.
Sewell's Self-Examination before Confirmation	1s. 6d.
— Readings for a Month Preparatory to Confirmation	4s.
— Preparation for the Holy Communion	3s.
— Readings for Every Day in Lent	5s.
— History of the Early Church	4s. 6d.
Cotton's Short Prayers and Helps to Devotion for School-boys	1s. 6d.

Mental and Moral Philosophy.

Bacon's Essays, with Annotations. By R. Whately, D.D. 8vo.	10s. 6d.
— — with Notes. By T. Markby, M.A.	1s. 6d.
— Advancement of Learning, with Notes, by T. Markby, M.A.	2s.
Elements of Logic, by R. Whately, D.D. 8vo. 10s. 6d. crown 8vo.	4s. 6d.
Stebbing's Abridgment of Mill's System of Logic	*Just ready.*
Elements of Rhetoric, by R. Whately, D.D. 8vo. 10s. 6d. crown 8vo.	4s. 6d.
Morell's Handbook of Logic	2s.
— Introduction to Mental Philosophy, 8vo.	12s.
— Fichte's Contributions to Mental Philosophy	5s.
— Elements of Psychology. Part I.	7s. 6d.
Thomson's Outline of the Necessary Laws of Thought	5s. 6d.

Civil Law and Political Science.

Marcet's Rich and Poor, for Children and Young Persons	1s.
— Willy's Holidays, or Conversations on Government	2s.
— Conversations on Political Economy	7s. 6d.
Twiss's Law of Nations as Independent Political Communities. Part I. The Rights and Duties of Nations in time of Peace, 8vo.	12s.
Part II. The Rights and Duties of Nations in time of War, 8vo.	18s.
Humphreys's Manual of British Government in India	2s. 6d.
Sandars's Institutes of Justinian, with Translation Notes, 8vo.	15s.

Principles of Teaching, &c.

Tate's Philosophy of Education	6s. 6d.
Floyd on Teaching Reading, Writing, and Arithmetic, 8vo.	2s. 6d.
Raumer's Life and System of Pestalozzi, by J. Tilleard, 8vo.	3s.
Robinson's Manual of Method and Organisation	6s. 6d.
Ross's Teacher's Manual of Method	3s. 6d.
Gill's Text-Book of Education, Method, and School Management	2s. 6d.
Stow's Training System of Education	6s. 6d.
Sullivan's Papers on Popular Education and School-Keeping	2s.
Walford's Handybook of the Civil Service	4s. 6d.
Potts's Liber Cantabrigiensis, 2 vols.	each 4s. 6d.
Pycroft's Course of English Reading	5s.
Arnold's Manual of English Literature	10s. 6d.
Lake's Book of Object Lessons	1s. 6d.
Hall's Guide to the Three Services, Civil, Naval, and Military	3s. 6d.

The Greek Language.

Kennedy's Greek Grammar, Sixth Edition	4s. 6d.
— Materials for Translation into Greek Verse	5s. 6d.

Collis's Pontes Classici, No. II. Greek	3s. 6d.
— Ponticulus Græcus, to accompany above	1s.
— Praxis Græca. PART I. Etymology, 2s. 6d.; PART II. Syntax	4s.
— Chief Rules of Greek Accentuation, with Exercises and Examination Papers. PART III. of Praxis Græca	3s.
— Exercises in Greek Tragic Senarii	4s. 6d.
— Chief Tenses of the Greek and Latin Irregular Verbs, 8vo.	each 1s.
Wilkins's Manual of Greek Prose Composition	7s. 6d.
Key, for Tutors and Students, price 2s. 6d.	
— Elementary Exercises in Greek Prose Composition	4s. 6d.
Key, for Tutors only, price 2s. 6d.	
— Progressive Greek Delectus for Schools	4s.; Key 2s. 6d.
— Progressive Greek Anthology, for Schools	5s.
Major's Elementary Praxis of Greek Composition	2s. 6d.
Valpy's Elements of Greek Grammar, 8vo.	6s. 6d.
— Greek Delectus, by Rev. J. T. White, M.A.	4s.; Key 2s. 6d.
Moody's New Eton Greek Grammar	4s.
Hall's Principal Roots of the Greek Tongue	5s.
Yonge's New English-Greek Lexicon, 4to.	21s.
Liddell and Scott's Larger Greek English Lexicon	31s. 6d.
— Smaller Greek-English Lexicon	7s. 6d.
Webster and Wilkinson's Greek Testament, 2 vols. 8vo.	44s.
Vol. I. the Four Gospels and Acts of the Apostles, 20s.	
Vol. II. the Epistles and the Apocalypse, 24s.	
Bloomfield's Greek Testament, with English Notes, 2 vols. 8vo.	48s.
— College and School Greek Testament, fcp.	7s. 6d.
— Lexicon to the Greek Testament	7s. 6d.
Robinson's Greek and English Lexicon of the New Testament, 8vo.	18s.
Major's Anthon's Homer's Iliad, Books I. to III. with English Notes	4s. 6d.
Linwood's Sophoclis Tragœdiæ superstites, 8vo.	16s.
Sophoclis *Œdipus Rex*, *Œdipus Coloneus*, and *Antigone*, edited by Brasse, 5s. each; Sophoclis *Philoctetes*, edited by Burges, 5s.; Sophoclis *Ajax* and *Electra*, edited by Valpy	each 5s.
Xenophon's Anabasis, Books I. and II. edited by D. B. Hickie, LL.D.	3s. 6d.
— Memorabilia of Socrates, by the same	3s. 6d.
— Expedition of Cyrus, by Rev. J. T. White, M.A.	7s. 6d.
Parry's Reges et Heroes, from Herodotus	3s. 6d.
Wilkins's Olnythiacs of Demosthenes, crown 8vo.	4s. 6d.

The Latin Language.

White and Riddle's Latin-English Dictionary, royal 8vo.	42s.
White's Advanced Latin-English Dictionary, medium 8vo.	*In the press.*
— Junior Latin-English Dictionary, square post 8vo.	*In preparation.*
Riddle's Diamond Latin-English Dictionary	4s.
— Complete Latin-English and English-Latin Dictionary, 8vo.	21s.
Separately—English-Latin Dictionary, 7s.; Latin-English Dictionary,	15s.
— Young Scholar's Lat.-Eng. and Eng.-Lat. Dictionary, square 12mo.	10s. 6d.
Separately—Latin-English Dictionary, 6s.; English-Latin Dictionary,	5s.
Riddle and Arnold's English-Latin Lexicon, 8vo.	25s.
Ebden's English-Latin Dictionary, abridged from the above	10s. 6d.
Kennedy's Elementary Latin Grammar	3s. 6d.
— Child's Latin Primer	2s.
— Latin Vocabulary, on Etymological Principles	3s.
— First Latin Reading-Book, or Tirocinium	2s.
— Second Latin Reading-Book, or Palæstra Latina	5s.

Kennedy's Materials for Translation into Latin Prose	6s.
— Examples of Latin Prose-Style, 12mo.	4s. 6d.; Key 7s. 6d.
Moody's New Eton Latin Grammar, 2s. 6d.; Accidence, separately	1s.
Collis's Pontes Classici, No. I.	3s. 6d.
— Ponticulus Latinus, to accompany above	1s.
— Praxis Latina	Part I. 2s. 6d.; Part II. 3s.
Miller's Elementary Latin Grammar	5s.
— Smaller Latin Grammar, abridged from the above	2s. 6d.
Hall's Principal Roots and Derivatives of the Latin Language	4s. 6d.
Hamilton's Analytical Latin Grammar	3s. 6d.
Zumpt's Larger Latin Grammar, translated by Dr. L. Schmitz, 8vo.	14s.
Bradley's Lessons in Continuous Latin Prose	5s.; Key 5s. 6d.
— Latin Prose Exercises	3s. 6d.; Key 5s.
Major's King's College School Latin Exercises	2s. 6d.
— Passages from the Spectator, for Translation into Latin	3s. 6d.
Wilkins's Manual of Latin Prose Composition	5s.
Key, for Tutors and Students, price 2s. 6d.	
— Elementary Latin Prose Exercises	4s. 6d.; Key 5s.
— Notes for Latin Lyrics, Sixth Edition	4s. 6d.
— Latin Anthology, for Junior and Middle Classes	4s. 6d.
Hiley's Elements of Latin Grammar	3s.
— First Progressive Latin Exercises	1s. 6d.
Valpy's Elements of Latin Grammar	2s. 6d.
— Latin Delectus, by the Rev. J. T. White, M.A.	2s. 6d.; Key 3s. 6d.
White's Progressive Latin Reader	3s. 6d.
— Latin Grammar, price 2s. 6d.; or separately, Accidence, 1s.; Eton Grammar, 1s. 9d.; Second Grammar	1s. 6d.
Yonge's New Latin Gradus	9s.; or with Appendix 12s.
— Classified Dictionary of Latin Epithets	3s. 6d.
Rapier's Introduction to Latin Verse	3s. 6d.; Key 2s. 6d.
Walford's Exercises in Latin Elegiac Verse	2s. 6d.; Key 5s.
— — Second Series, with Grammar of Latin Poetry	2s. 6d.
Johns's Eton Latin Verse-Book, or *Ductor in Elegias*	2s. 6d.
Edwards's Progressive Exercises in Latin Lyrics	3s.
Yonge's Horace, I. Odes and Epodes, 3s.; II. Satires and Epistles	3s. 6d.
Girdlestone and Osborne's Horace	7s. 6d.
Kennedy's School Edition of Virgil	*Nearly ready.*
Major's Anthon's Æneid of Virgil, with English Notes	5s.
Pycroft's Virgil's Æneid, Bucolics, and Georgics	7s. 6d.; without Notes 3s. 6d.
Valpy's Virgil's Bucolics, Georgics, and Æneid	7s. 6d.; without Notes 3s. 6d.
Bradley's Troy Taken, or Second Book of Virgil's Æneid	2s. 6d.
Parry's Origines Romanæ, from Livy	4s.
Bradley's Cornelius Nepos, by the Rev. J. T. White, M.A. 12mo.	3s. 6d.
— Eutropius's Epitome, by the Rev. J. T. White, M.A.	2s. 6d.
— Fables of Phædrus, by the Rev. J. T. White, M.A.	2s. 6d.
— Ovid's Metamorphoses, by the Rev. J. T. White, M.A.	4s. 6d.
Valpy's Ovid and Tibullus, as used at Eton, with English Notes	4s. 6d.
Isbister's School Edition of Cæsar's Commentaries	3s. 6d.
Hickie's First Five Books of Livy	8s. 6d.
White's Germania and Agricola of Tacitus	4s. 6d.
White's Cicero's Cato Major and Lælius	3s. 6d.

The French Language.

Sadler's Stepping-Stone to French Pronunciation	1s.
Hamel's French Grammar and Exercises, by Lambert	5s. 6d.; Key 4s.
Collis's Short French Grammar for Classical Schools	3s. 6d.

Simmons and Durlac's Exercises on Dr. Collis's French Grammar	*Just ready.*
Brasseur's Practical Exercises on French Phraseology	3s. 6d.
Albites' How to Speak French, by Achille, Seventh Edition, revised	5s. 6d.
— Instantaneous French Exercises	1s.; Key 2s.
Contanseau's Practical French and English Dictionary	10s. 6d.
— Pocket French and English Dictionary	5s.
— First Step in French	2s. 6d.
— French Grammar, Fifth Edition, remodelled and enlarged	5s.
Key to the Exercises in Contanseau's 'First Step' and 'French Grammar'	3s.
Contanseau's Guide to French Translation	3s. 6d.; Key 3s. 6d.
— Prosateurs et Poètes Français	6s. 6d.
— Précis de la Littérature Française	5s.
— Abrégé de l'Histoire de France	5s. 6d.
Ventouillac's French Poetry, with English Notes	2s.
Sewell's Contes Faciles, for the use of Little Girls	3s. 6d.
— Extraits Choisis des Auteurs Modernes, for Young Ladies' Schools	5s.
Stièvenard's Lectures Françaises, from Modern French Authors	4s. 6d.
De Poix-Tyrel's Grammar of Household Words, French and English	4s. 6d.

German, Italian, and Hebrew.

Wintzer's First German Book for Beginners	3s. 6d.
De Poix-Tyrel's Grammar of Household Words, English, German, French, and Italian	10s. 6d
— Grammar of Household Words, German and English	4s. 6d.
Bernays' Compendious German Grammar	3s. 6d.
— German Exercises, 3s. 6d.; German Examples	3s.
— Schiller's Mary Stuart, 2s.; Maid of Orleans, 2s.; William Tell	2s.
Just's New German Reading Book	3s. 6d.
Kalisch's Hebrew Grammar, with Exercises. Parts I. and II. price 12s. 6d. each; and Key to the Exercises in Part I. price	5s.
Blackley and Friedlander's Practical German Dictionary	*In the press.*

Hindustani and Sanskrit.

Williams and Mather's Practical Hindustani Grammar and Hindustani Selections	5s.
Mather's Glossary, Hindustani and English, to the New Testament and Psalms	7s.
Benfey's Sanskrit-English Dictionary	*In preparation.*

THE valuable Dictionary of the late Professor H. H. WILSON has long been out of print, while the dictionaries now in course of publication are planned on a scale which will require many years for their completion, and are designed for the use of advanced scholars only. A dictionary of moderate size is therefore much needed for those who are studying the language for the sake of its literature, and for the purpose of applying it in Comparative Grammar. The present work, which is intended to supply this want, will give only those words which are found in the actual literature of the language, all words being omitted which rest only on the authority of grammarians, while even those roots, which have not yet been authenticated, are marked by a cross+. And, although, to keep the work within moderate limits, words occurring in the Vedic literature only will not be included, none will be omitted which are necessary for reading the Law-book of Manu, the Bhagavadgítá, the Hitopadesa, and almost all the standard works hitherto published. A list of these works will be inserted in the preface. Each word will be given in the original Sanskrit and in a Latin transliteration, and of the principal words short etymological notices will be furnished, the cognate words from the Greek, Latin, and Teutonic languages being given in all cases where their relationship with the Sanskrit is well established.

CLASSICAL SCHOOL-BOOKS.

A PROGRESSIVE GREEK ANTHOLOGY, for the use of Schools. By the Rev. H. Musgrave Wilkins, M.A. Fellow of Merton College, Oxford. Second Edition. 12mo. 5*s.*

HOMER'S ILIAD, Books I. to III. with Notes and Homeric Glossary of Anthon. Edited by Dr. Major, Head Master of King's College School. 12mo. 4*s.* 6*d.*

SOPHOCLIS TRAGŒDIÆ superstites, recensuit et brevi Annotatione instruxit Gulielmus Linwood, M.A. Ædis Christi apud Oxonienses nuper Alumnus. Editio Altera. 8vo. 16*s.*

SOPHOCLES, from the Text of Brunck and others, with English Notes, Examination Questions, &c. *Œdipus Rex*, *Œdipus Coloneus*, and *Antigone*, edited by Dr. Brasse, post 8vo. 5*s.* each: *Philoctetes*, edited by G. Burges, M.A. 5*s.* *Ajax* and *Electra*, edited by the Rev. F. Valpy, 5*s.* each.

The OLYNTHIACS of DEMOSTHENES, from the Text of Bekker; with Historical and Critical Prefaces and copious English Notes. Edited, for the use of the Upper Classes in Schools and of University Students, by the Rev. H. M. Wilkins, M.A. Crown 8vo. 4*s.* 6*d.*

A COMPLETE LATIN-ENGLISH and ENGLISH-LATIN DICTIONARY, for the use of Colleges and Schools. By the Rev. J. E. Riddle, M.A. 8vo. 21*s.*

Separately { The ENGLISH-LATIN DICTIONARY, 7*s.*
The LATIN-ENGLISH DICTIONARY, 15*s.*

RIDDLE'S YOUNG SCHOLAR'S LATIN-ENGLISH and ENGLISH-LATIN DICTIONARY; being a School Abridgment of his Complete Latin Dictionary. Square 12mo. price 10*s.* 6*d.*

Separately { The LATIN-ENGLISH DICTIONARY, 6*s.*
The ENGLISH-LATIN DICTIONARY, 5*s.*

A NEW LARGE LATIN-ENGLISH DICTIONARY. By the Rev. J. T. White, M.A. of Corpus Christi College, Oxford; and the Rev. J. E. Riddle, M.A. of St. Edmund Hall, Oxford. Imperial 8vo. 42*s.*

An ADVANCED LATIN-ENGLISH DICTIONARY, for the use of Upper Form Pupils and University Students. Abridged from the above by the Rev. John T. White, M.A. Medium 8vo. [*In the press.*

A JUNIOR LATIN-ENGLISH DICTIONARY, for the use of Junior Pupils and the Lower Forms in Grammar Schools. Abridged from the above by the Rev. John T. White, M.A. Square post 8vo. [*In preparation.*

It is intended to publish two Abridgments of the large Latin Dictionary of the Rev. J. T. White and the Rev. J. E. Riddle, for the use of students in the Universities and in public and private schools.

The plan of both these Abridgments corresponds in all essential features with that of the original work. A complete Lexicon of the Latin language from its earliest to its latest stages is, however, needed chiefly by persons whose reading extends beyond what are called the Classical Authors. The larger Abridgment, in medium octavo, is designed, therefore, to illustrate fully the writers which are usually read in the Public Schools and Universities; and, accordingly, words and meanings of words, found only in Glossaries, Ecclesiastical Writers, and such Authors as Ammian, Petronius, Apuleius, &c. have been omitted. The number of examples in each article is necessarily diminished; but enough has been left in every instance to elucidate the various modes of construction and the several shades of meaning through which each word has passed. Most of the proper names have been retained, and the etymology has been compared with various works on the subject, which have appeared, whether wholly or in part, within the last two years.

This larger Abridgment is now in the press, and is expected to be ready in the Autumn. The smaller Abridgment, for the use of Schoolboys and Junior Pupils, will be sent to press after the foregoing has been published.

London: LO Paternost